

ELEMENTARY BUILDING GEOMETRY

BY

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LONDON

EDWARD ARNOLD & CO.

First published 1935

Printed in Great Britain by
Butler & Tanner Ltd., Frome and London

FOREWORD

This book has been designed to meet the needs of the young building student specializing for the first time in a branch of geometry particularly suited to his calling.

The subject-matter covers the first year syllabuses of the various Local, County, and National Examination Boards.

To keep the price of the book within reasonable limits and at the same time omit nothing of importance in the subject-matter, was the greatest problem the author had to face in the preparation of the book.

Another vital question that arose was concerned with *originality*. In an old subject like geometry this question is an important one. It is almost impossible to be *original* in dealing with the subject itself, the only scope for originality lying in its **presentation and application**.

It has been assumed that the students for whom this volume intends to cater, have received instruction in some branch of Technical Drawing, either in the Manual Instruction Centre, the Elementary or Secondary Day School, the Junior Technical School, or the Evening Continuation Classes. Consequently, chapters on Drawing Instruments, Angles, and general elementary principles have been omitted, and the space they would have occupied devoted to what the author considers to be more important constructions.

The writer takes this opportunity of recording his thanks to Mr. E. W. Haddon of the Leicester College of Art and Technology for his helpful suggestions, and to Mr. A. Parker of the Hull Technical College for his help in correcting the proofs.

G. A. H.

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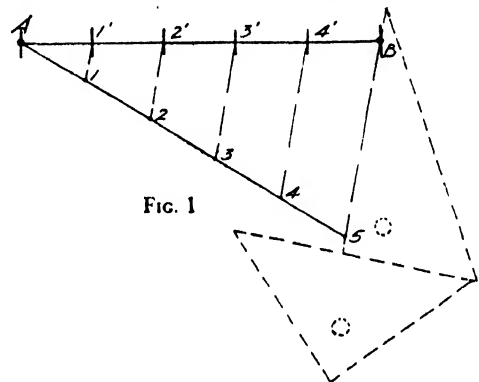
CHAPTER I

Scales

THERE are two types of scales in general use, namely *plain* and *diagonal*. In building work, plain scales are used to a greater extent than diagonal scales. From a good plain scale, it should be possible to take off two different units of length, as yards and feet, feet and inches, miles and furlongs, and so on. From a diagonal scale, three different units of length can be measured, as yards, feet and inches, inches, tenths and hundredths, and so on.

The scale (which, of course, must be accurate) should always appear on the drawing sheet, so that any measurement inadvertently omitted can be readily ascertained without recourse to the rule.

Many maps are drawn to a scale of $\frac{1}{2}$ in. to the mile. This means that half an inch in length on the drawing represents one mile of actual length. It should be obvious then that an error of only $\frac{1}{100}$ th of an inch on the scale means a difference of many yards on the actual length. It requires a skilled person to transfer measurements from the rule directly on to the paper, so this method of setting out a scale is not to be recommended for the beginner.



Division of Lines

Before proceeding to draw scales, let us divide a line AB (Fig. 1) into any number of equal parts (say five) by the geometrical method.

Draw the line A 5 at any angle to AB, and on this line mark off five equal divisions with the dividers or compasses in such a manner that the last point 5 falls almost underneath B. Join B to 5. Now place the set squares in the position shown, and by sliding the 60° set square along the 45° set square, the parallel lines 4' 4', 3' 3' . . . can be drawn. The line AB should now be accurately divided into five equal parts or units. This method of dividing lines into units is the one to be adopted in scale drawing.

Common Building Scales

In building there are three common scales, namely :

- (a) A scale in which $\frac{1}{8}$ in. represents 1 ft. (Described as the "eighth-inch scale.")
- (b) A scale in which $\frac{1}{2}$ in. represents 1 ft. (Described as the "half-inch scale.")
- (c) A full-size scale.

The first of these scales is used for the general lay-out of the building, i.e. for the plans, elevations and sections of the building as a whole. Drawings made to this scale show the positions of doors, windows, drains, manholes, etc., relative to the whole building.

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Drawings made to the half-inch scale are usually termed the "half-inch details." These details show fully the construction of various detached parts of the building, irrespective of the remainder, e.g. the half-inch details may include an elaborate gable, the staircase, the various windows, doors, etc. Drawings made to this scale are sufficiently large and clear to enable the builder to see exactly what is required.

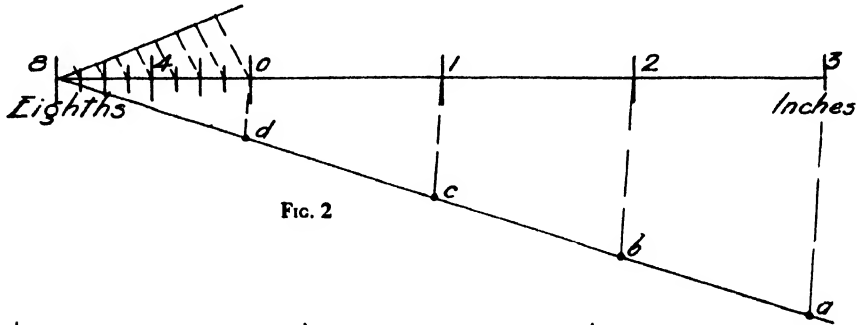


FIG. 2

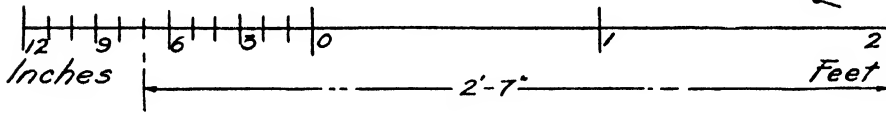


FIG. 3

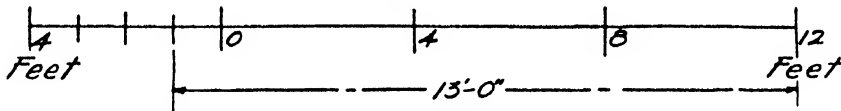


FIG. 4

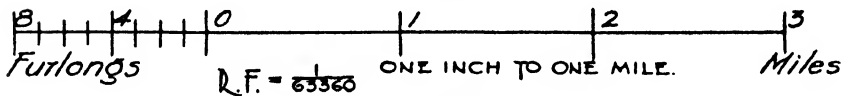


FIG. 5

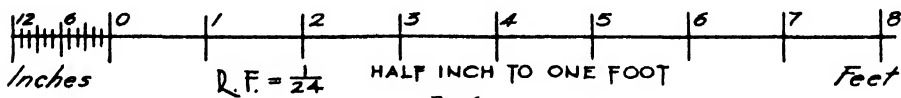


FIG. 6



FIG. 7

The full-size scale will, of course, enable the designer to draw the various units actually the same size and shape as they are intended to be in the completed building. Under this heading, all mouldings (other than standard moulds) will appear. The mason must have a pattern or templet before he can work the cornice mould of the building, and similarly the wood-cutting machinist requires the true shape of the moulded skirtings, picture rails, etc., in order that he can shape his cutters for the moulding machine or spindle. Drawings other than full size would be useless for these purposes.

Representative Fraction (R.F.)

The R.F. of a scale is equal to

$$\frac{\text{The unit of the scale}}{\text{The distance the unit represents}}$$

e.g. A scale of 1 in. to 1 ft. would have an R.F. of $\frac{1 \text{ in.}}{12 \text{ in.}}$ or $\frac{1}{12}$.

A scale of 1 in. to 1 yd. would have an R.F. of $\frac{1 \text{ in.}}{36 \text{ in.}}$ or $\frac{1}{36}$, and the R.F. of a scale of 1 in. to 1 mile would be $\frac{1}{1760 \times 3 \times 12} = \frac{1}{63360}$

Various Plain Scales.

EXAMPLE. (Fig. 2.) *To draw a full-size scale to read up to 4 in. and to measure eighths of an inch.*

Draw line 8 3 accurately 4 in. in length. By the method shown in Fig. 1 divide the line into four equal units, and the extreme left division into eight equal units in the same manner. Draw short perpendicular lines at the points thus obtained and figure the scale as shown. The correct figuring of a scale is very important. Hard and fast rules cannot be laid down on this point, as the figuring will vary according to the type of scale.

It is fairly safe to assume, however, that the figure 0 is placed at the point of intersection of the long and short units, and the other figures appropriately inserted to left and right of the 0. The scale must be neat and workmanlike, but remember that accuracy is of far more importance than elaboration.

EXAMPLE. (Fig. 3.) *To draw a scale of $1\frac{1}{2}$ in. to 1 ft. to read to 3 ft. and to show separate inches. R.F. = $\frac{1}{8}$.*

Draw a line 12 2 $4\frac{1}{2}$ in. (i.e. $3" \times 1\frac{1}{2}"$) long, and divide it into three equal parts for the feet units. Divide the extreme left unit into twelve equal parts for the inch measurements, and complete the scale as shown. A length of 2 ft. 7 in. is shown on the figure.

EXAMPLE. (Fig. 4.) *To draw a scale the R.F. of which is $\frac{1}{48}$.*

R.F. is $\frac{1}{48} \therefore 1 \text{ in. represents } 48 \text{ in. or } 4 \text{ ft.}$

Divide a line of indefinite length into one-inch units: each unit will represent 4 ft. The extreme left division is now divided into four equal parts for the separate feet. A length of 13 ft. is marked off.

Various scales are shown in Figs. 5, 6 and 7.

Diagonal Scales

If a scale of $\frac{1}{8}$ in. to 1 ft. was required (Fig. 9) we could draw a line of indefinite length, and divide it into eighth-inch units. If, however, we desired that the scale should be constructed in such a manner that separate inches could be taken off, the extreme left unit would have to be sub-divided into twelve parts; this is almost an impossible task. It is possible though to arrive at this result in a different manner. Refer to Fig. 8: abc is a right-angled triangle, ab being $\frac{1}{8}$ in. and the angle abc the right angle. bc is any length, but it is

divided accurately into twelve equal units. de, fg , etc., are lines drawn parallel to ab , therefore the triangles clm, cjk, cfg , etc., are similar. Now any two corresponding sides of similar triangles are in the same proportion to one another as any other two corresponding sides of the same triangles, e.g. in Fig. 8 $ck : kj :: cg : gf :: cm : ml$ and so on. As bc is divided into twelve equal units, and $ab = \frac{1}{8}$ in., de will be $\frac{1}{8}$ ths of ab , i.e. $\frac{1}{8}$ ths of $\frac{1}{8}$ in. But ab represents

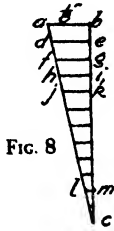


FIG. 8

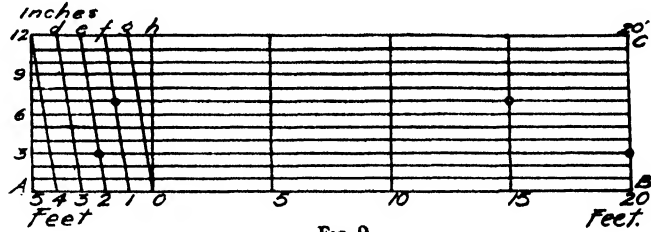


FIG. 9

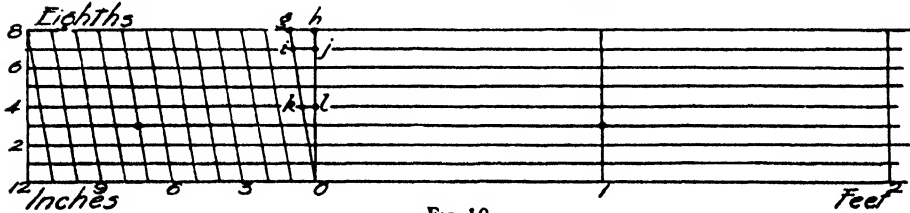


FIG. 10

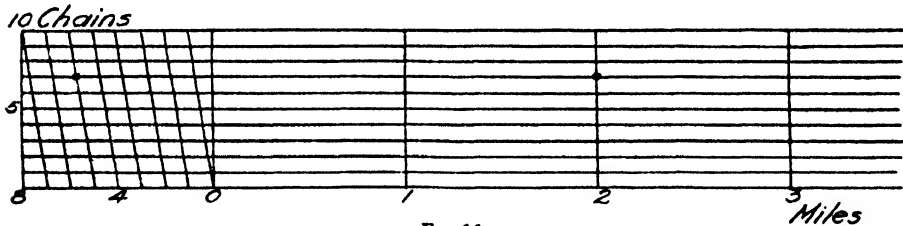


FIG. 11

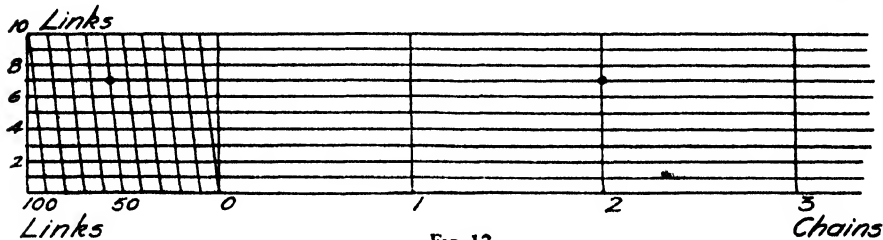


FIG. 12

1 ft. $\therefore de$ represents 11 in. Similarly fg represents $\frac{1}{8}$ ths of ab , i.e. 10 in., and lm represents 2 in.

This method of dividing a line is adopted in drawing diagonal scales.

Now to proceed with the scale. As $\frac{1}{8}$ in. has to represent 1 ft., 5 ft. will be represented by $\frac{5}{8}$ in. Divide a line AB (Fig. 9) into $\frac{5}{8}$ in. units, and AO into five equal units: this would constitute a scale of $\frac{1}{8}$ in. to 1 ft. showing separate feet. Draw the line $A 12$ perpendicular to AB , and mark on this line twelve equal units with the dividers (these units may be any convenient length). Complete the rectangle $ABC 12$ with the perpendicular divisions drawn

from 5, 10, 15, etc., and the parallel horizontal lines. Draw the line 4 12, and parallel to this 3d, 2e, etc. Figure the scale as shown.

The right-angled triangle of Fig. 8 corresponds to the triangle *ohg* of Fig. 9. If we have a distance of say 22 ft. 3 in. to take off, place one divider point on 20 and the other point on 2 on the line AB; this length represents 22 ft. Now follow the line 20 20' and the line 2e until we arrive at the horizontal line from 3; the intersection of these lines will be the points required, as the small length of the horizontal line crossing the right-angled triangle *ohg* is $\frac{3}{12}$ ths of 1 ft. or 3 in. Another measurement of 16 ft. 7 in. is shown on the scale.

EXAMPLE (Fig. 10.) *To draw a scale of $1\frac{1}{2}$ in. to 1 ft. to show feet, inches, and eighths of an inch.*

Divide a line, 12 2 of indefinite length into the required number of $1\frac{1}{2}$ -in. units, and the line 12 0 into twelve units. A plain scale of $1\frac{1}{2}$ in. to 1 ft. showing inches is thus obtained. From 12, mark off eight vertical units, draw the horizontal and vertical parallel lines, and complete the scale as shown. The right-angled triangle *ohg* is divided into eight similar triangles; but *gh* represents 1 in., \therefore *ij* will represent $\frac{7}{8}$ in., *kl* $\frac{6}{8}$ in., and so on. Two points 1 ft. $7\frac{3}{8}$ in. apart are shown.

EXAMPLE. (Fig. 11.) *To draw a diagonal scale of 1 in. to 1 mile to show miles, furlongs and chains.*

The main horizontal divisions are 1 in. in length. As there are eight furlongs in a mile, and ten chains in a furlong, the horizontal divisions on the left will represent furlongs, and the vertical divisions chains. Two points 2 mls. 5 fur. 7 ch. apart are shown.

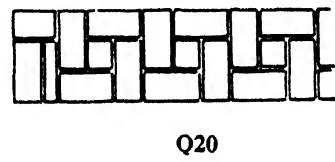
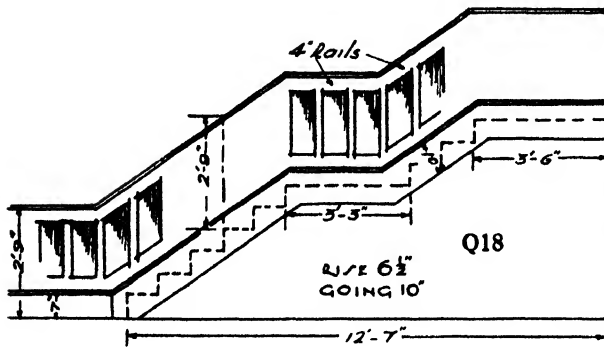
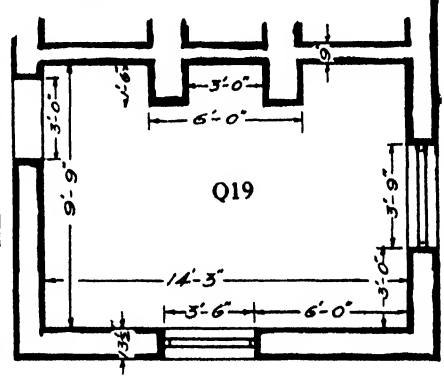
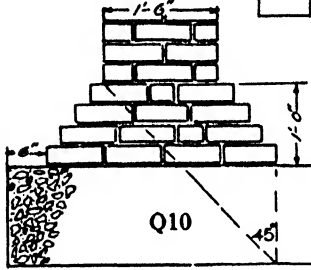
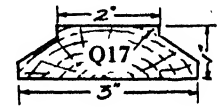
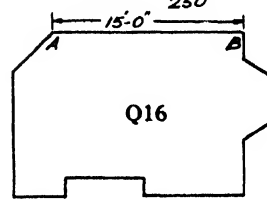
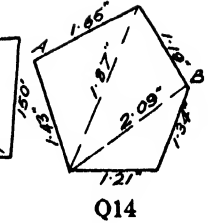
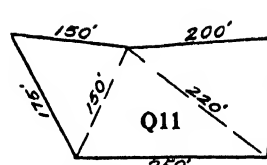
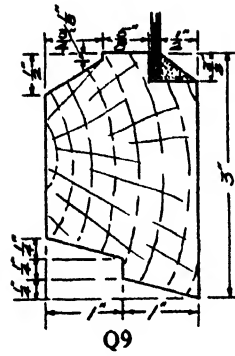
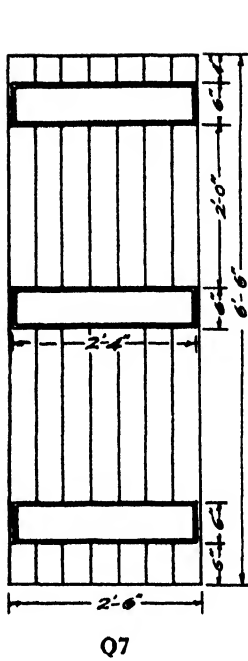
EXAMPLE. (Fig. 12.) *To construct a scale of 1 in. to 1 chain to show links.*

As there are 100 links in 1 chain, the small horizontal divisions on the left multiplied by the vertical units must equal one hundred. Ten by ten will be convenient. The construction is similar to that of the preceding scales. Two points 2 ch. 57 links apart are marked.

(Note. If the words "Inches" and "Tenths" are substituted for the words "Chains" and "Links" on the lower horizontal line, and the word "Hundredths" for "Links" on the upper left-hand corner, the scale would be a full-size one, reading to the second decimal place.)

EXERCISES

1. Draw a line $3\frac{1}{2}$ in. long and divide it into seven equal units by a geometrical method. (Ref. Fig. 1.)
2. Divide the extreme left unit of the line in Q 1 into four equal parts.
3. Divide the line of Q 1 in the ratio of 2 : 3 : 4.
4. If the line of Q 1 represents a length of 3 ft. 6 in., lengthen it so as to make it 4 ft. 8 in. (no calculations allowed).
5. Assuming the line in Q 1 represents a length of 2 ft. 6 in., divide and figure it so that it forms a scale from which separate inches can be taken.
6. Assuming the face of a brick to measure 9" \times 3", draw to a scale of 1 in. to 1 ft. a stretching course (i.e. a row of bricks placed end to end) 4 ft. 6 in. long, after first drawing the scale



7. Draw a scale of $\frac{3}{4}$ in. to a foot, and from it set out the back elevation of the ledged door shown in the figure. The width of the door must be geometrically divided.

8. A lawn 100 ft. long and 56 ft. wide is surrounded by a path 8 ft. wide on the long sides. The width of the path on the short sides is unknown, but the diagonals of the grass lawn are also the diagonals of the lawn and path together. Set out the lawn and path to a scale of $\frac{1}{32}$ in. to 1 ft., and measure the width of the path on the short sides. What is the length of the diagonals of the lawn?

9. A section through the bottom rail of a sash is shown in the figure. Set out the rail full size, and complete the detail by inserting all dimensions.

10. The diagram shows a section through the foundation of a 2 brick wall. Set out the detail to a scale of $1\frac{1}{2}$ in. to a foot.

(Note.—Draw fine single lines of all mortar joints first, and assume these lines to be the centre lines of the double lines forming the completed joints.)

11. The outline of a plot of land is given, the measurements being in feet. Draw the plot to a scale of 1 in. to 100 ft., and ascertain the area of the plot in square yards, by any method.

12. Draw a scale one-eighth full size to show inches and quarter-inches, and to read up to 4 ft.

13. Construct a full-size scale in such a manner that sixty-fourths of an inch may be taken off. From the scale, plot a rectangle $2\frac{3}{8}" \times 1\frac{3}{4}"$. What is the length of the diagonal?

14. Construct a scale suitable for setting out the figure to the dimensions given (the figure is not drawn to scale). What is the distance from A to B correct to the second place of decimals?

15. Construct a scale the R.F. of which is $\frac{1}{32}$. The scale must be available for reading yards, feet and inches.

16. Draw the scale from which the given diagram was plotted, and tabulate the lengths of all the sides. $AB = 15$ ft.

17. The section shows an architrave mould designed for a villa residence. To a scale of half full size, design (a) a $7" \times 1"$ skirting board, (b) a $2\frac{1}{2}" \times \frac{3}{4}"$ picture rail, (c) a $2" \times \frac{7}{8}"$ sash bar for the same building in order that the mouldings conform to the given design.

18. A portion of a stairs string and dado panelling is shown in the diagram. Draw a half-inch diagonal scale (i.e. $\frac{1}{2}$ in. = 1 ft.) and from it set out the full details of the stairs and panelling as shown.

19. Construct a scale of $\frac{1}{2}$ in. to a foot and make use of it in setting out the given sectional plan.

20. Draw a scale $\frac{1}{8}$ full size to show quarter inches. Make use of the scale in drawing the portion of $1\frac{1}{2}$ ft. brick wall in Flemish Bond. The plan of a whole brick plus one mortar joint measures $9" \times 4\frac{1}{2}"$.

21. A line 4 in. long represents 3 miles. Construct a scale on this line to show miles, furlongs and chains. Draw a triangle of sides 2 mls. 1 fur. 2 ch., 1 ml. 5 fur. 4 ch., and 1 ml. 7 fur. 7 ch. respectively.

CHAPTER II

The Triangle

Before commencing actual problems on the construction of triangles, it is necessary to note :

(a) The three angles of any triangle together equal 180° .

(b) Triangles drawn in the same segment of a circle (with the chord as base) have vertical angles of the same magnitude, e.g. in Fig. 13 AB is the base of all triangles, and $\angle AEB = \angle ADB = \angle ACB$.

(c) Triangles drawn on the same base with similar altitudes are equal in area, e.g. in Fig. 14 $\triangle ABC = \triangle ABD = \triangle ABE = \triangle ABF$ because FC is parallel to the base AB, and consequently all the triangles have the same altitude.

You will be already acquainted with the fact that an equilateral triangle has three equal sides, an isosceles triangle has two equal sides, and a scalene triangle has no sides equal.

Triangles from Simple Data

We will now proceed to deal with a few elementary problems on the construction of triangles.

EXAMPLE. (Fig. 15.) *A triangular courtyard is bounded by three walls the lengths of which are 55 ft., 50 ft. and 45 ft. respectively. Make a plan of the yard to a scale of $\frac{1}{2}$ in. to 10 ft.*

This problem could be re-written thus : "Draw a triangle of sides $2\frac{3}{4}$ in., $2\frac{1}{2}$ in., and $2\frac{1}{4}$ in."

Draw one side AB $2\frac{3}{4}$ in. long. With compass point at A, radius $2\frac{1}{4}$ in., strike an arc to intersect another arc struck from B with a $2\frac{1}{2}$ in. radius. ABC is the required triangle.

EXAMPLE. (Fig. 16.) *Two trees T and T₁ on the bank of a stream are 25 yds. apart. Another landmark T₂ is sighted on the opposite bank, and it is found that the angle TT₁T₂ is 42° and the angle T₁TT₂ is 57° . What is the width of the stream?*

Draw a line TT₁ 25 yds. long to a suitable scale. With the protractor set off angles of 42° and 57° at T₁ and T respectively. The point T₂ is thus located. The distance T₂A is the width of the stream.

EXAMPLE. (Fig. 17.) *On a triangular site, it was found to be impossible to measure the three sides of the site on account of an intervening building. In the survey of the site, two sides were measured and found to be 100 ft. and 75 ft. respectively, and the angle between these sides was 40° . Draw the triangle.*

To any convenient scale, draw AC 100 ft. in length. At A, set off with the protractor an angle of 40° , making this arm of the angle 75 ft. long. Complete the triangle ABC as shown.

EXAMPLE. (Figs. 18 and 18A.) *A yard is roughly a triangle in shape. Certain obstructions prevent direct measurements being made when the yard is surveyed for paving. It is found, however, that the perpendicular CD is 25 ft., the angle at A is 46° , and the angle at B is 40° .*

We have, therefore, to plot a triangle given the two base angles, and the perpendicular from the vertex to the base.

Draw two parallel lines AB and FE 25 ft. apart (Fig. 18A). At any point C on FE set off angles of 46° and 40° in opposite directions. The arms of these angles will intersect the line ADB at points A and B. These points are the other two corners of the triangle.

EXAMPLE. (Fig. 19.) *The fence around a triangular plot measures 155 yds.: one side measures 65 yds. and a perpendicular from this side to the opposite angle is 30 yds.*

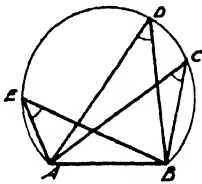


FIG. 13

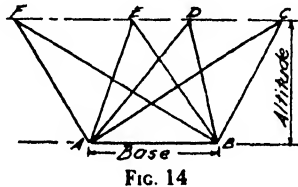


FIG. 14

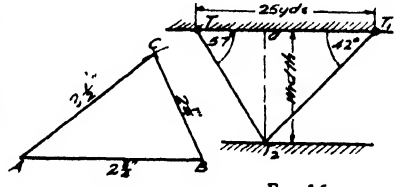


FIG. 15

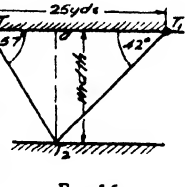


FIG. 16

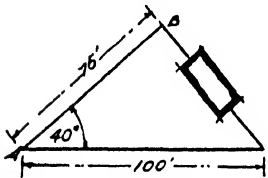


FIG. 17

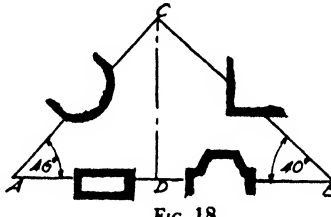


FIG. 18

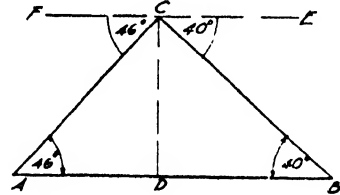


FIG. 18A

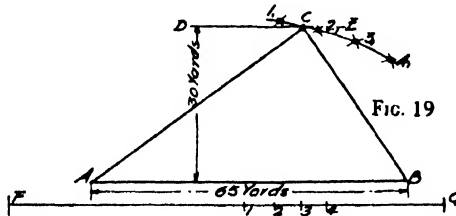
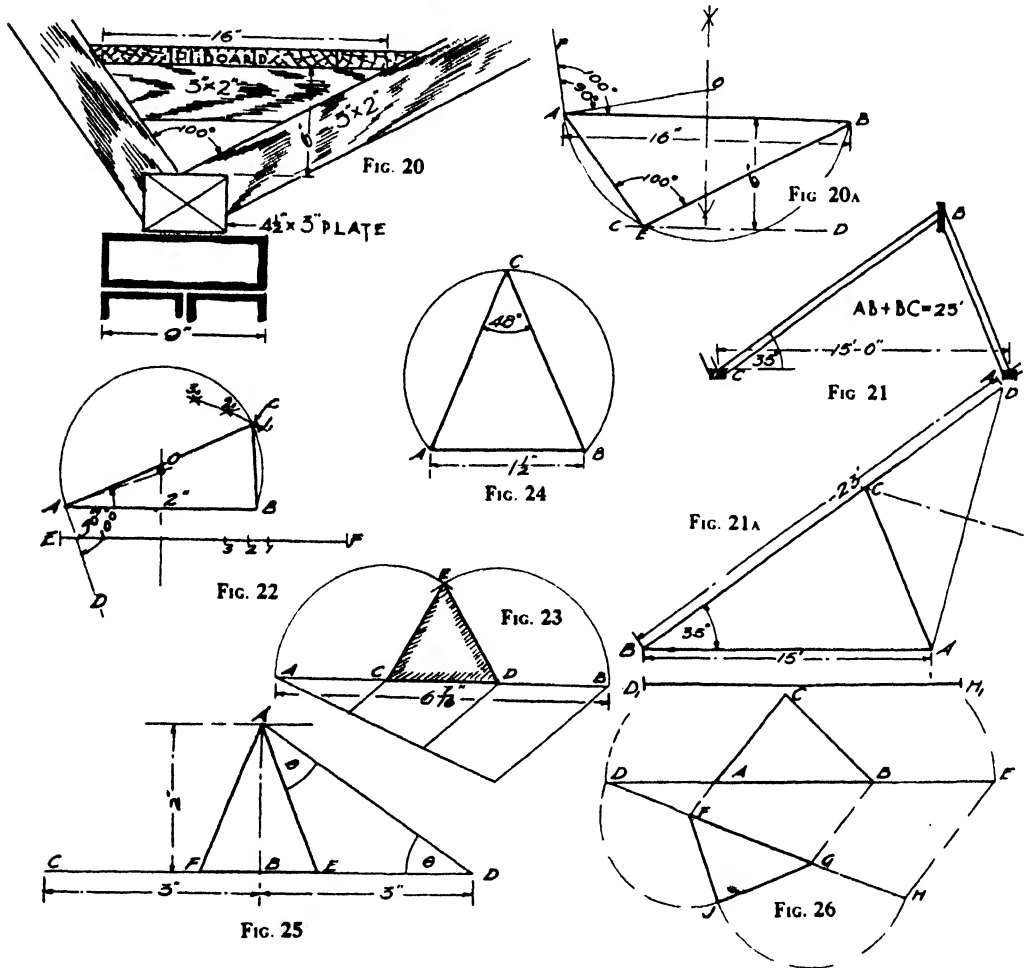


FIG. 19

To plot this site, draw AB 65 yds. long to an appropriate scale. If the base is 65 yds., then the other two sides together equal 90 yds. Make FG = 90 yds., and mark several points 1 2 3 . . . on this line near its centre. With radius F1 strike an arc from A to intersect another arc struck from B with a radius of G1: the intersection of these arcs is 1_1 . Again, with centre A radius F2, strike an arc to intersect another arc struck from B with radius G2: the intersection is at 2_1 . Repeat this construction for the points 3_1 , 4_1 and more, if necessary, and trace a fair curve through the points thus obtained. Now draw a line DE parallel to AB, and 30 yds. distant from AB. The intersection of this line and the freehand curve is the vertex required. (Note.—The freehand curve (if continued) represents the path of the vertices of all triangles of base AB, and perimeter 155 yds.)

EXAMPLE. (Figs. 20 and 20A.) *The angle between the rafters of two roof surfaces is 100° . There is a gutter formed between the roofs as shown in Fig. 20, the top edge of the gutter bearer being 16 in. long and the perpendicular distance from the upper edge of the bearer and the upper face of the wall plate is 6 in.*

To set out the detail, we must first consider the data available for the principal triangle. This triangle has one side (say the base) of 16 in. length, a vertical height of 6 in., and a vertical angle of 100° .



It has been stated previously that all angles in the same segment of a circle are equal (Fig. 13); the problem under consideration introduces this theorem. To draw a segment to contain an angle of 100° on a base or chord 16 in. long, draw AB (Fig. 20A) 16 in. in length (to a suitable scale) and at one end of the line A make an angle BAF equal to 100° . At right angles to FA draw AO to intersect the bisector of AB at O; O is the centre of the segment AEB that contains 100° , or, the angle between the lines drawn to A and B from any point on the curve will be 100° . Draw CD parallel to AB and 6 in. distant from the latter;

the point E will be the vertex of the required triangle. From this triangle build up the detail.

EXAMPLE. (Figs. 21 and 21A.) *In a roof of the type shown in Fig. 21, the two rafters AB and BC together measure 23 ft. The distance from centre to centre of the supporting walls is 15 ft. and the rafter BC is pitched at 35°. We are, therefore, given the base, one base angle, and perimeter of the triangle ABC.*

Make AB (Fig. 21A) 15 ft. long to a suitable scale, the angle ABD 35°, and BD 23 ft. long. We have now to "bend" BD at a certain point C such that D will coincide with A. Join AD and bisect it. This bisector projected to C (on DB) will be the required vertex of the triangle, as ADC is now an isosceles triangle with AC and DC the equal sides. But $BC + CD = 23$ ft., $\therefore BC + CA$ will equal 23 ft.

We will now construct a few triangles from data supplied, and apply the construction in the Exercises which follow.

EXAMPLE. (Fig. 22.) *Construct a triangle given the perimeter (say 5 in.), the base (2 in.) and the vertical angle (70°).*

Draw AB 2 in. long, and on this line construct a segment to contain 70°, i.e. make $\angle BAD = 70^\circ$ and $\angle DAO = 90^\circ$, O being the centre of the segment. Now proceed as in Fig. 19 making EF equal to the (perimeter — base), or 3 in. in length. The third angle of the triangle is thus located at C.

EXAMPLE. (Fig. 23.) *Construct an equilateral triangle of perimeter $6\frac{1}{4}$ in.*

Make AB $6\frac{1}{4}$ in. long and divide it into three equal parts by the method described in the chapter on Scales (Fig. 1). With C as centre, radius CA, and with D as centre, radius DB describe arcs to intersect at E. CDE is the required triangle.

EXAMPLE. (Fig. 24.) *Construct an isosceles triangle of $1\frac{1}{2}$ in. base and 48° vertical angle.*

There are several methods of drawing this triangle. The simplest would be to draw the base AB $1\frac{1}{2}$ in. long, and at each end of the line to make angles of $\frac{180^\circ - 48^\circ}{2}$ (or 66°) as the three angles together must equal 180°. The method shown in Fig. 24 is to construct a segment to contain an angle of 48° on a $1\frac{1}{2}$ in. chord, and then to make $AC = BC$.

EXAMPLE. (Fig. 25.) *Construct an isosceles triangle of altitude 2 in. perimeter 6 in.*

Draw CD equal in length to the given perimeter (6 in.) and the perpendicular bisector BA equal to the given altitude (2 in.). Join AD. Make angle DAE = angle BDA, and BF = BE. FAE is the required triangle.

(Note.—As $\angle \theta = \angle \theta$, the triangle EDA is isosceles, i.e. $DE = AE$. But $BE + ED =$ half perimeter, therefore $BE + EA =$ half perimeter, and $FE + EA + AF$ equals whole perimeter.)

EXAMPLE. (Fig. 26.) *Construct a triangle similar to a given triangle ABC, but having a perimeter equal to D_1H_1 .*

With A as centre, AC as radius, locate D on AB continued. Find point E in a similar manner. DE is then equal to the perimeter of the triangle ABC.

Draw DH (equal in length to D_1H_1) at any angle to DE. Join EH, and parallel to this line, draw BG and AF. Now construct the required triangle with sides respectively equal to DF, FG, and GH.

EXERCISES

1. Part of a building site has to be fenced off in the form of a triangle, one side of which measures 30 yds. The area of the plot must be 300 sq. yds., and the fence must be as short as possible. Draw the plot to a scale of 1 in. to 10 yds.

2. A grass plot of sides 65 yds., 55 yds. and 45 yds. respectively, is surrounded by paths 10 yds. wide. Three other paths of the same width join the surround at the angular points, on the outside of the triangle, these paths having the same direction as the bisectors of the angles of the triangle. Set out the plot and paths to a scale of $\frac{3}{8}$ in. to 10 yds. (Ref. Fig. 15.)

3. A simple Vee gutter made of 1 in. boards is shown in the diagram. Draw the section to the measurements given. Scale $\frac{1}{4}$ full size.

4. The trestles for a flat draughting-table are made from $3'' \times 1\frac{1}{4}''$ timber, the elevations being as shown. Set out the trestle to a scale of 1 in. to 1 ft.

5. Set out the wall crane to the data provided. Scale $\frac{3}{8}$ in. to 1 ft.

6. A King-post roof truss is shown in the diagram. The King-post BD is 10 ft. long and the principal rafters BA, BC make angles of 35° with the tie-beam AC. The struts DF and DE bisect the rafters. Draw the truss to a scale of $\frac{1}{8}$ in. to 1 ft.

7. Set out a line diagram of the section of the roof shown in the figure, using a scale of $\frac{1}{4}$ in. to 1 ft. (Ref. Fig. 24.)

8. Set out the ridge tile, the distance $a + b + c$ being 2 ft. 10 in. Scale 3 in. to 1 ft. (Ref. Fig. 25.)

9. A leaded light is in the form of an equilateral triangle of 6 ft. 4 in. perimeter. All the comes forming the design, inside the light, are of the same length. Set out the light to a scale of 1 in. to 1 ft. (Ref. Fig. 23.)

10. A building of width 32 ft. terminates with a gabled wall, the apex angle of which is 70° . The copings fixed on the sloping sides of the gable are not of equal length, but together they measure 54 ft. The distance from the apex to the ground level is 40 ft. Draw the gable to a scale of $\frac{1}{8}$ in. to 1 ft. (Ref. Fig. 22.)

11. In the given steel roof truss, ED = 10 ft., angle BED = angle BDE = 55° . Angle FBE = angle GBD = angle FAE = angle GCD = 25° . FE and GD bisect the rafters AB and BC respectively. Set out the truss to a scale of $\frac{1}{8}$ in. to 1 ft.

12. A triangular templet ABC has to be made to the following dimensions: $AB + BC + CA = 16$ ft., $AB = 6$ ft., and the perpendicular from AB to C is 3 ft. 6 in. Draw the outline of the templet to a scale of $\frac{3}{8}$ in. to 1 ft. (Ref. Fig. 19.)

13. The angle between two walls is 120° . It is required to fit a corner cupboard in the angle to the dimensions shown on the diagram. Set out a single line plan of the cupboard to a scale of 1 in. to 1 ft. (Ref. Fig. 20A.)

14. It is required to fix a fence parallel to the longest side of a triangular plot of ground of sides 55 yds., 48 yds. and 63 yds., in such a position that the total length of fencing of the smaller triangular plot must be 120 yds. Draw the original plot, and the position of the required fence to a scale of $\frac{1}{2}$ in. to 10 yds. What is the area of the smaller plot? (Ref. Fig. 26.)

15. The total length of the boundary of a triangular site is 9 ch. 60 links. One side measures 4 ch. 23 links, and the angle between this side and one of the adjacent sides is 37° . Plot the site to a scale of $\frac{1}{8}$ in. to 1 ch. (Ref. Fig. 21A.)

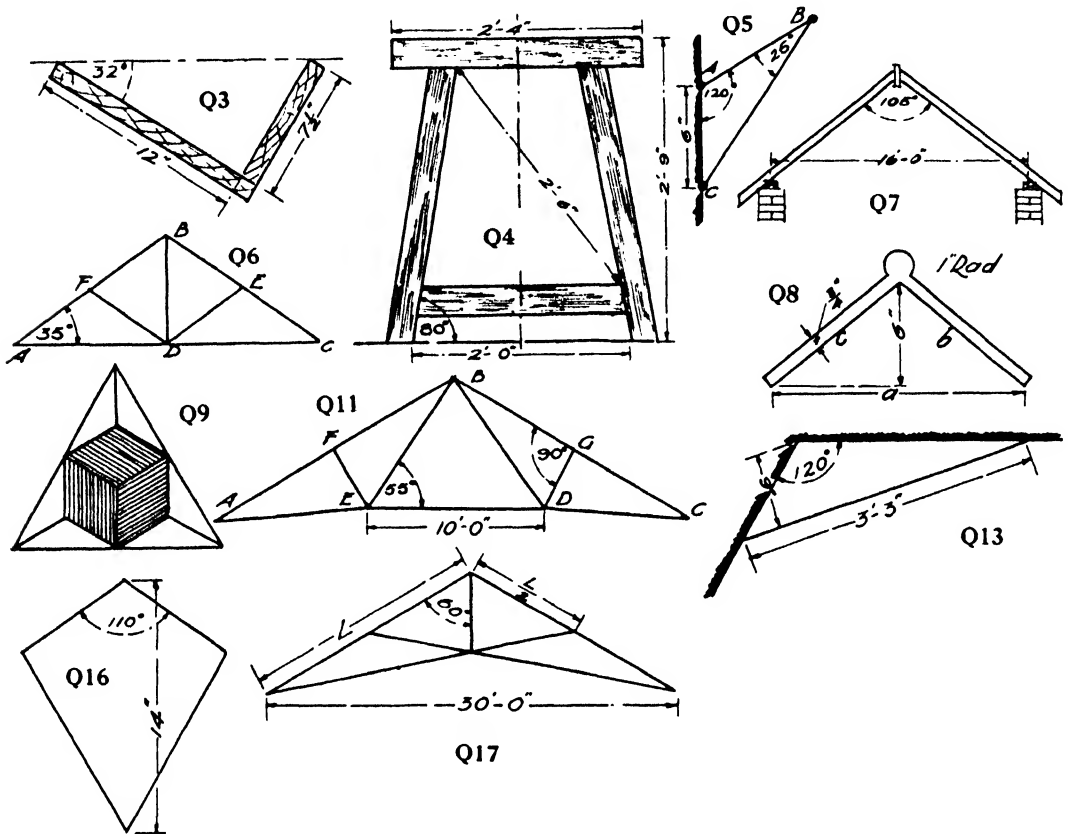
16. The total length of the sides of a kite-shaped piece of lead is 38 in. : other particulars are shown in the diagram (not drawn to scale). Set out the piece to $\frac{1}{4}$ scale. (Ref. Fig. 21A.)

17. Set out the roof truss to the data provided. Scale $\frac{1}{8}$ in. to 1 ft.

18. A wood triangular templet is made from laths 55 ft. long, the sides of the triangle being in the ratio of 3 : 5 : 7. Draw the templet to a scale of $\frac{1}{8}$ in. to 1 ft. and tabulate the magnitudes of the three angles.

The Right-Angled Triangle

The right-angled triangle is more frequently used in building perhaps than any other type of triangle; therefore, from a geometrical point of view, it demands our greatest attention.

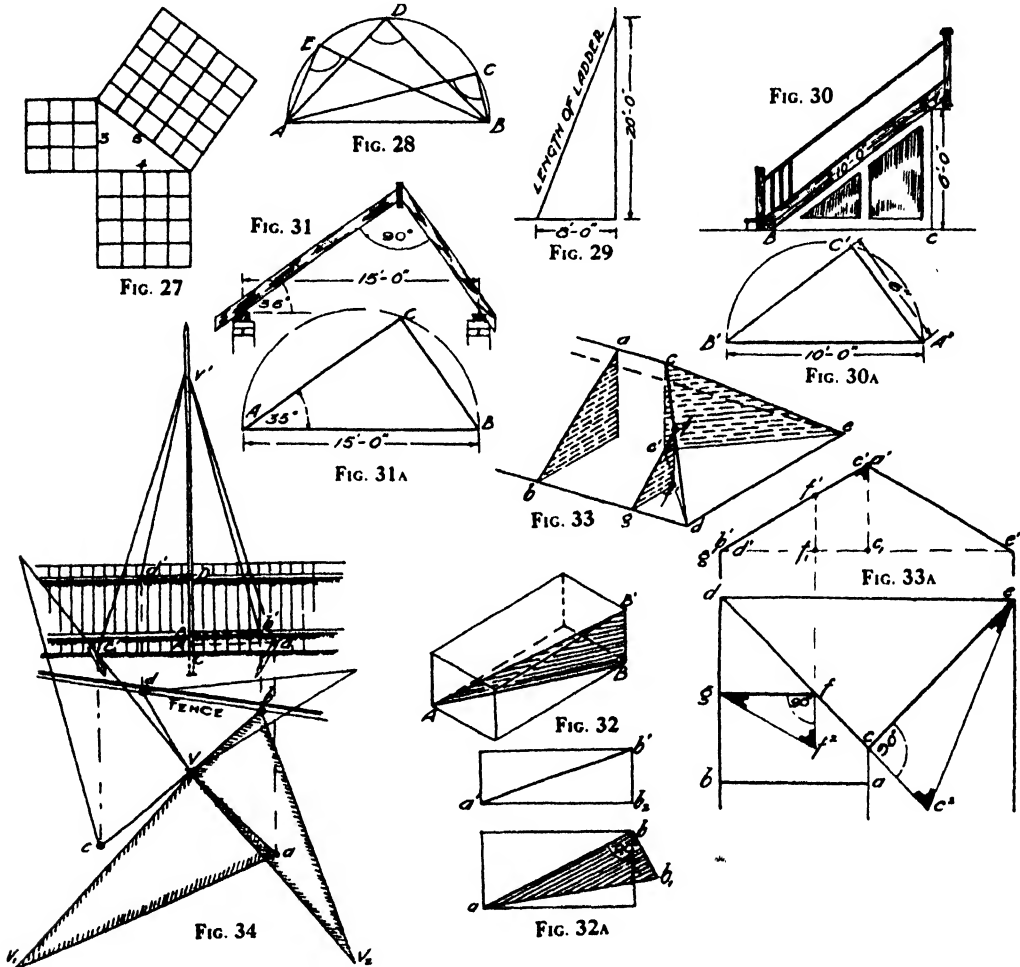


You have probably been taught that "The square on the hypotenuse of a right-angled triangle equals the sum of the squares on the other two sides." This statement, which is known as Proposition 47 of the First Book of Euclid or the Theorem of Pythagoras, can be demonstrated easily by drawing a triangle of sides 3, 4, and 5 units long respectively, erecting squares on each side and dividing these squares into smaller squares of one unit size as shown in Fig. 27. You will have, of course, twenty-five squares in the larger square (the square on the hypotenuse) and sixteen and nine small squares in the squares on the other two sides

respectively. Thus $16 + 9 = 25$; also you will find the triangle is truly right-angular. Any right-angled triangle may be subjected to a similar test.

Another important fact concerning the right-angled triangle is, "All angles contained in a semicircle are right angles." Thus in Fig. 28 $\angle ACB = \angle ADB = \angle AEB = 90^\circ$, because AEDCB is a semicircle.

EXAMPLE. (Fig. 29.) Find the length of ladder required to reach a vertical height of 20 ft. The ladder foot to be not more than 8 ft. from the vertical.



Draw a right-angled triangle of sides 20 ft. and 8 ft. respectively. The hypotenuse will be the length required.

(By calculation:—Length of ladder = $\sqrt{20^2 + 8^2} = 21.54$ ft., say 21 ft. 6 in.)

EXAMPLE. (Figs. 30 and 30A.) A spandrel frame has to be made for the underside of a stair string as shown in Fig. 30. Set out the templet for the spandrel.

Make A'B' (Fig. 30A) 10 ft. long, and on it erect a semicircle. With centre A', radius

6 ft., cut the semicircle at C' , and complete the triangle. (*Note.*—The angle at C' must be a right angle as $A'C'B'$ is a semicircle.)

EXAMPLE. (Figs. 31 and 31A.) *To set out the roof shown in Fig. 31, when the span, pitch of rafters on one side, and angle (90°) at the ridge are given.*

(Fig. 31A.) On AB (15 ft. long), to an appropriate scale, describe a semicircle. At A set off an angle to 35° to intersect semicircle at C . ACB is the single line diagram from which the complete detail can be drawn.

EXAMPLE. (Figs. 32 and 32A.) *To find the length of the longest diagonal that can be contained in a rectangular prism.*

Fig. 32A is the plan and elevation of the prism and diagonal ab , and Fig. 32 a pictorial view.

You will notice first that neither the plan nor the elevation of the diagonal in Fig. 32A shows the *true length* of the diagonal, as ab is inclined to the eye in a downward direction, and the elevation $a'b'$ is sloping *away* from the eye. If we refer to Fig. 32 we find that AB is the plan of the diagonal, and forms the base of a right-angled triangle ABB' . Now ab in Fig. 32A corresponds to AB in Fig. 32, and the height of the elevation b_1b' corresponds to BB' . Therefore, given the plan and elevation of the diagonal, we have sufficient data to draw the right-angled triangle, the hypotenuse of which will be the true length of the diagonal. Make bb_1 equal to b_1b' and perpendicular to ab ; then ab_1 is the real length of the diagonal, as the triangle abb_1 now corresponds to the right-angled triangle ABB' in Fig. 32.

Figs. 33 and 33A show the application of the right-angled triangle to a simple hipped roof. Fig. 33 is a pictorial and Fig. 33A an end view and plan of the same roof.

The length of the common rafter ab is taken direct from the end elevation, and is therefore $a'b'$. The hip rafter ce (Fig. 33) is the hypotenuse of the right-angled triangle ecc' .

In Fig. 33A ce is the plan and $c'e'$ the elevation of the hip rafter; neither of these lines shows the *true length* of the hip, but ce (Fig. 33A) is the same length as $c'e$ (Fig. 33), or, in other words, ce (Fig. 33A) may represent the base of a right-angled triangle, the vertical height of which is equal to $c'e_1$. (All heights are taken from eaves level.) Construct a right-angled triangle ecc^2 on ce making cc^2 equal to the vertical height $c'e_1$; the true length of the hip will be ec^2 .

The length of the jack rafter fg (Fig. 33) is obtained in a similar manner. In Fig. 33A fg is the plan and $f'g'$ the elevation of the jack rafter: the perpendicular height of f is $f'f_1$, therefore construct a right-angled triangle gff^2 with fg as base and f_1f' vertical height: f^2g is the true length of the jack rafter, as this line is equivalent to the hypotenuse of the right-angled triangle $f'gf$ in Fig. 33.

The side bevels for the base and head of the rafters are shown.

EXAMPLE. (Fig. 34.) *The diagram shows the plan and elevation of a wireless mast held in position by four wire stays: the stays are anchored at varying heights. The true length of the stay wires are required.*

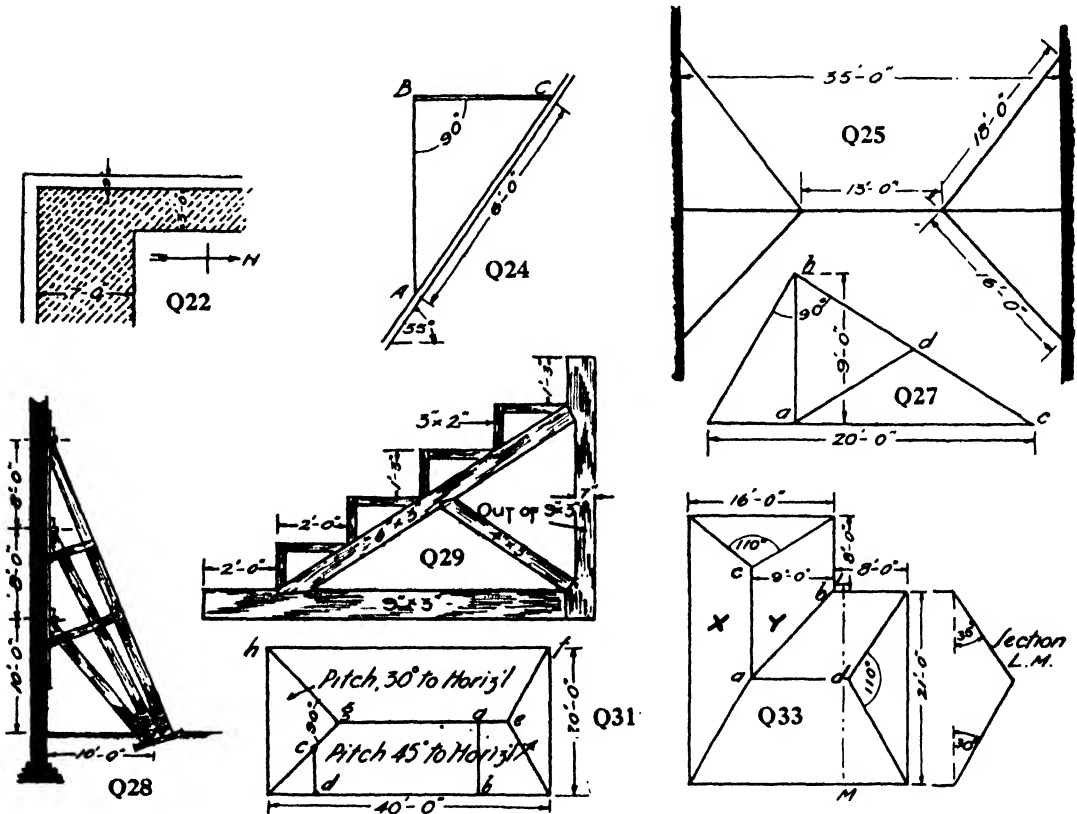
Each stay will form the hypotenuse of a right-angled triangle, the base of which appears on plan, and the vertical height in elevation. E.g., aV is the plan and $a'V'$ the elevation of one of the stays. To find the true length of this stay, draw the right-angled triangle aVV_1 , such that VV_1 equals the vertical height of V' above a' , i.e. $V'A$. The true length of the stay is aV_1 . Notice that this right-angled triangle would fit vertically under the stay wire, V_1 coinciding with V' , a with a' and V with A . Repeat this construction for the other stays making $VV_2 = V'B$ and so on.

EXERCISES (Contd.)

19. What height would a 10-yds. ladder reach on a vertical wall if the foot of the ladder were 8 ft. from the wall base? Check your graphical answer by a calculated answer.

20. In plan, the diagonal of a rectangular building measures 56 ft., and one side of the building measures 20 ft. Plot the plan to a scale of $\frac{1}{2}$ in. to 10 ft. What is the length of the long side of the building? (Ref. Fig. 30A.)

21. A level courtyard is surrounded by a wall 6 ft. high. The courtyard is rectangular measuring 43 ft. by 18 ft., the longer sides running due E. and W. What is the length of the shadow cast by the wall on the ground when the sun is in the west and its rays make 32°



with the horizontal? Draw the plan of the courtyard and shadows to a scale of 1 in. to 20 ft.

22. The diagram represents the angle of two walls 7 ft. high, and the shadow cast by the walls on the ground. What is the angle of the sun's rays with the ground? From what direction is the sun shining? (Ref. Fig. 32.)

23. You have to set out the lines for the trenches prior to building two walls meeting at a right angle; the walls are 50 ft. and 73 ft. long respectively. The only setting out gear available consists of a cord of indefinite length, a few stakes, and a two-foot rule. How would you proceed with the marking out? Illustrate by sketches. (Ref. Fig. 27.)

24. A side view of a dormer window is shown in the figure. Set out the lead for the cheek to the particulars supplied. No laps need be shown. (Ref. Fig. 31A.)

25. The diagram represents the section of the walls of two buildings supported by flying shores. Set out the outline to a scale of 1 in. to 10 ft.

26. A symmetrical brick gable has an apex angle of 90° and a width of 25 ft. Find the total length of coping required. Compare your answer with the mathematical solution. (Ref. Fig. 28.)

27. An outline diagram of a roof truss is shown. The strut ad bisects the rafter bc . Set out a single line diagram of the truss to a scale of $\frac{1}{8}$ in. to 1 ft. (Ref. Fig. 28.)

28. The three raking shores shown in the diagram temporarily support the face of the building. Find the total length of $9" \times 9"$ stuff required for the shores by means of a scale drawing, and check your answer by calculations.

29. A standard for a temporary staging is shown. Draw the standard to a scale of $\frac{1}{2}$ in. to 1 ft. (Note:—All ironwork has been omitted.)

30. What is the length of the longest diagonal that can be contained in a rectangular prism $2\frac{1}{4}" \times 1\frac{1}{2}" \times \frac{3}{4}"$? (Ref. Fig. 32A.)

31. The plan of a hipped roof given. Find the length of the common rafter ab , the jack rafter cd , and the hip rafters ef and gh . Scale 1 in. to 10 ft. (Ref. Fig. 33A.)

32. A rectangular room is 21 ft. long, 14 ft. wide and 10 ft. in height. An electric bulb is suspended on a flex 2 ft. long from a point in the ceiling 6 ft. from the long side of the room, and 9 ft. from the short side. Find graphically the distance of the bulb from the four lower corners of the room. Scale 1 in. to 5 ft. (Ref. Fig. 34.)

33. In the sketch of the hipped roof, the ridges ca and da are assumed to be horizontal. Draw the roof to a scale of 1 in. to 10 ft., and find the lengths of all the hip rafters, and the valley rafter ab . What are the slopes of the two roof surfaces marked X and Y?

34. Draw in fine lines an equilateral triangle abc of $1\frac{1}{2}$ in. side and find its centre: from the centre draw lines to each corner of the triangle. The diagram now represents the plan of a flag staff (centre of triangle) and three wire stays connecting the staff to the corners of the triangle, drawn to a scale of $\frac{1}{4}$ in. to 3 ft. The staff is 36 ft. high, and the stays are fastened at a point 6 ft. from the top of the staff. The bottom ends of the wires a , b and c are anchored at points 1 ft., 2 ft. 6 in., and 3 ft. 9 in. respectively above ground level. Draw an elevation of the staff and stays, and find the total length of wire required for the latter. (Ref. Fig. 34.)

CHAPTER III

Polygons

A polygon is usually regarded as a plane figure with more than four straight sides. The names of the polygons having from five to ten sides are as follows :

Five sides	Pentagon
Six „	Hexagon
Seven „	Heptagon
Eight „	Octagon
Nine „	Nonagon
Ten „	Decagon

A *regular* polygon has all its sides equal, and all its angles the same magnitude.

Hexagon and Octagon

The hexagon and octagon are perhaps the most frequently used regular polygons in building work, and incidentally are the easiest to construct, as both can be drawn with the set squares. Two methods of drawing the regular hexagon are shown in Figs. 35A and 35B.

EXAMPLE. (Figs. 35A and 35B.) *To draw a regular hexagon on side AB.*

Method 1 (Fig. 35A). At A and B draw two lines of indefinite length with the 60° set square, the latter occupying the positions shown in the diagram. Take the length AB on the compasses or dividers, and mark off with this length BC and AF. Now reverse the set square, draw CD and FE, and again mark off length AB. Complete the hexagon. Under no circumstances must you measure the sides of the polygon with the rule as accuracy cannot be guaranteed by this method.

Method 2. (Fig. 35B.) With a radius equal to the given side AB describe a circle. Commencing at any point A on the circumference step off the radius around the circumference. You will find that six steps will bring you back to A. Connect the points AB, BC, and so on.

The octagon is constructed in a similar manner to the hexagon except that the 45° set square is substituted for the 60° set square.

EXAMPLE. *To draw a regular octagon, given side AB.* (Fig. 36.)

Draw AH and BC with the 45° set square at the extremities of the given side AB. Set the compasses or dividers accurately to AB and mark off AH and BC. Complete the polygon by manipulating the 45° set square for the direction of the sides, the compasses or dividers, of course, remaining unaltered for the length of the sides.

In many problems connected with the hexagon and octagon, the length of side is not stated. Instead, the distance “across the flats” is given. This means the distance between any two parallel sides of the polygon as shown in Fig. 36 and is really the *diameter* of the polygon.

EXAMPLE. (Figs. 37A and 37B.) *To construct a regular hexagon given the diameter AB.*

Method 1. (Fig. 37A.) Describe a circle with AB as diameter. Now draw tangents to this circle with the 60° set square. (Two positions of the set square are indicated in Fig. 37A.)

Method 2. (Fig. 37B.) At A and B make angles of 30° with the set square, thus ascertaining the length of the side of the polygon AC. Now proceed as in Fig. 35A using the set square and dividers.

EXAMPLE. (Figs. 38A and 38B.) *To construct a regular octagon given the diameter AB.*

Method 1. (Fig. 38A.) Bisect AB at O, and with O as centre describe a circle with AB as diameter. Draw tangents to the circle with the 45° set square as shown.

Method 2. (Fig. 38B.) Draw a square CDEF with sides equal to AB, and locate its centre O. With radius equal to half the diagonal of the square, say DO, and centres C, D, E, and F describe arcs MH, GI, LJ, and KN. Complete the octagon as shown.

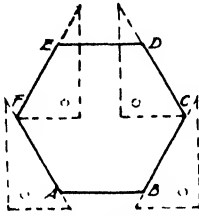


FIG. 35A

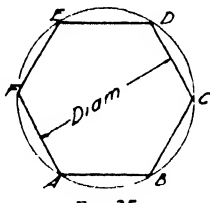


FIG. 35B

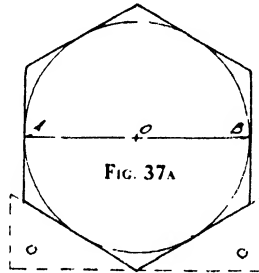


FIG. 37A

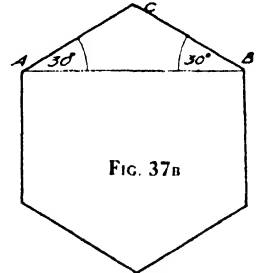


FIG. 37B

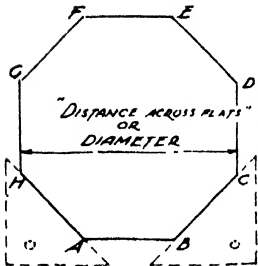


FIG. 36

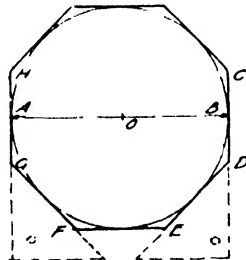


FIG. 38A

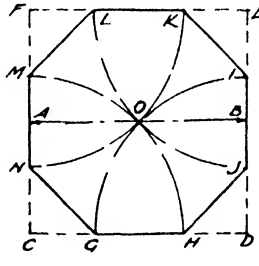


FIG. 38B

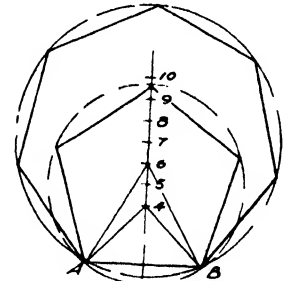


FIG. 39

Other Regular Polygons

EXAMPLE. (Fig. 39.) *To draw any regular polygon given the length of one side AB.*

Draw the bisector of AB. With AB as base construct triangles A4B and A6B with the 45° and 60° set squares respectively. Point 4 will be the centre of a circle of radius 4A that will exactly circumscribe a square of AB side. Similarly, point 6 will be the centre of a circle of radius 6A that will circumscribe a hexagon of side AB. It is then reasonable to suppose that the mid-point 5 between 4 and 6 will be the centre of a circle that will circumscribe a pentagon. Therefore with 5 as centre, and radius 5A describe a circle. You will find that by stepping off length AB around this circle, a regular pentagon will be obtained. To construct the other regular polygons make all units on the bisector of AB equal to 5 6. Point 7 will be the centre of a circle which will contain a heptagon, point 8 the centre of a circle circumscribing an octagon and so on. Great care must be taken in dividing the bisector into units as the smallest error will result in a much greater inaccuracy in the completed polygon.

Another method of drawing any regular polygon is shown in Fig. 40.

EXAMPLE. (Fig. 40.) *To draw a regular heptagon on side AB.*

Continue BA to C. With A as centre, AB as radius describe semicircle CDB. Divide (by trial) the semicircle into as many parts as the figure has sides, in this case seven. Join the second dividing point D to A. The intersection of the bisectors of AB and AD (point O) will be the centre of a circle which will pass through AB and D, and which will circumscribe the heptagon.

(*Note.*—Whatever number of sides the polygon contains always join the second point on the semicircle to the nearer end of the base.)

EXAMPLE. (Fig. 41.) *To draw a regular pentagon given the length of the diagonal EC.*

Consider the pentagon ABCDE. As O is the centre of the figure, angles AOB, BOC, etc., are equal and have a magnitude of $3\frac{1}{2}^\circ$ or 72° . As there are 180° in every triangle, angle EDO = $(180^\circ - 72^\circ) \div 2 = 54^\circ$. Angle EFD = 90° , \therefore angle DEF = $180^\circ - (90^\circ + 54^\circ) = 36^\circ$.

To construct the pentagon, at each end of the diagonal EC make angles of 36° , thus obtaining point D. Bisect ED and CD for point O the centre of the figure. With O as centre describe a circle to pass through E, D, and C. Step off DC five times around the circle.

EXAMPLE. (Fig. 42.) *To draw a heptagon ABCDEFG given the length of a long diagonal AD.*

Consider the heptagon in Fig. 42. If we can locate the centre of the heptagon O, we can describe the circumscribing circle of the polygon. Each of the angles AOB, BOC and COD is equal to $360^\circ \div 7$ \therefore angle AOD = $3(360^\circ \div 7) = 154\frac{2}{7}^\circ$, and angle OAD = angle ODA = $(180^\circ - 154\frac{2}{7}^\circ) \div 2 = 12\frac{2}{7}^\circ$.

To return to the problem, locate point O by setting off angles of $12\frac{2}{7}^\circ$ at each end of the diagonal AD. With O as centre, describe a circle to pass through A and D. Divide that part of the circumference of the circle lying between A and D into three equal units on one side of AD, and four on the other side, thus obtaining the seven angles of the heptagon. (This solution is not very satisfactory as $12\frac{2}{7}^\circ$ cannot be obtained accurately with the protractor. Further, the heptagon is one of the most difficult of the polygons to manipulate, as 360° is divisible by 5, 6, 8, 9, and 10 but not by 7.)

Irregular Polygons

Before we can plot any irregular polygon, certain data must be provided. This data may include (1) the lengths of the sides of the polygon, plus the lengths of the diagonals, (2) the lengths of the sides and the angles between them, (3) lengths of some of the sides and diagonals, and the magnitude of some of the angles.

When a polygon has to be drawn from written data it is advisable to sketch the polygon roughly, letter the corners, and mark on the sketch the particulars provided. This enables one to "build up" the scale drawing from the data.

EXAMPLE. (Fig. 43B.) *To draw a hexagonal plot of ground, ABCDEF irregular in outline, to the following measurements: AB = 225 yds., BC = 150 yds., CD = 125 yds., DE = $137\frac{1}{2}$ yds., EF = 125 yds., FA = $87\frac{1}{2}$ yds., AD = 175 yds., BD = 100 yds., AE = 175 yds.*

At first sight, this problem appears to be very confusing. On investigation, however, the assembling of the given measurements into the complete figure is quite a simple matter.

Roughly sketch a hexagon (Fig. 43A), letter the corners and mark the lines the lengths of which are given. Now we see at a glance how to proceed to assemble the parts of the polygon. Choose a convenient scale, say 1 in. to 100 yds., and commence by drawing triangle ABD. The other three triangles DBC, ADE and AEF are now plotted to complete the figure.

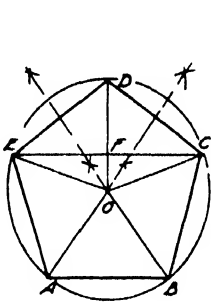


FIG. 41

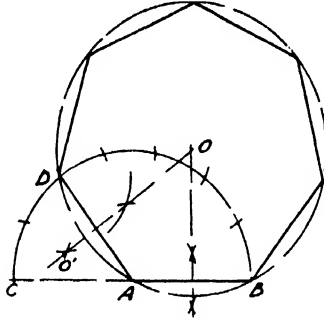


FIG. 40

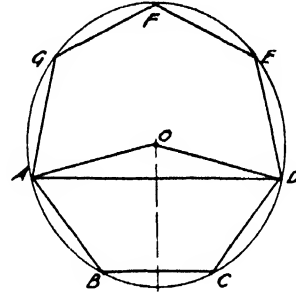


FIG. 42

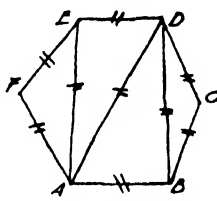


FIG. 43A

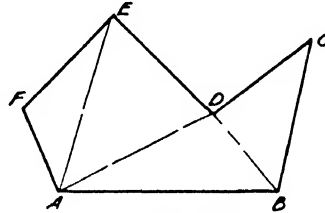


FIG. 43B

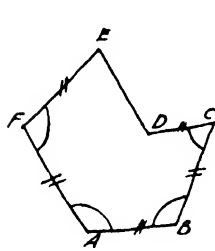


FIG. 44A

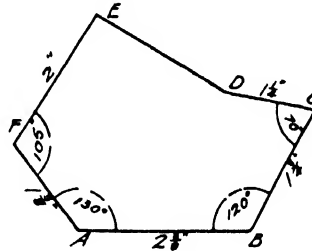
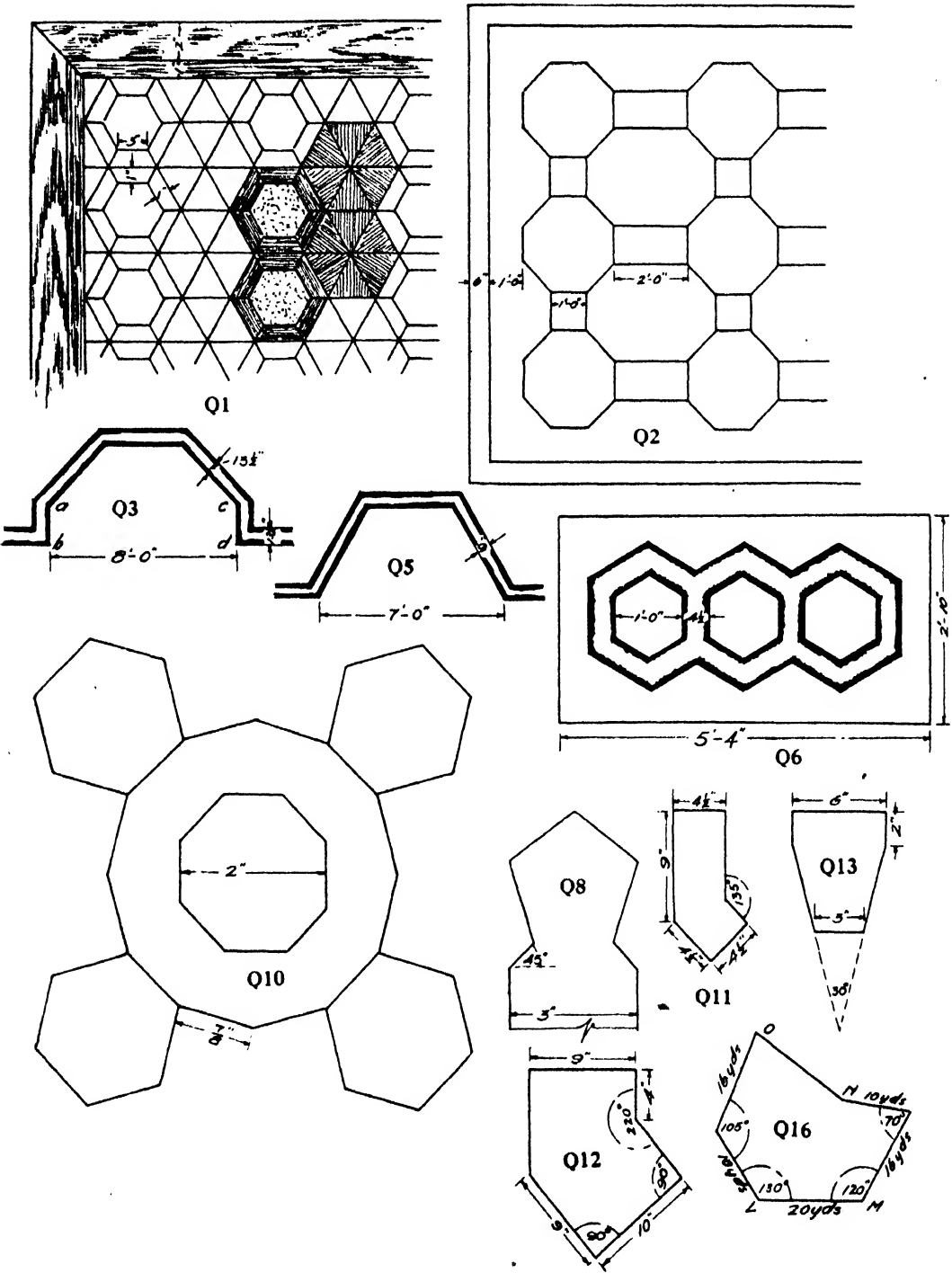


FIG. 44B

EXAMPLE. (Fig. 44B.) To draw an irregular hexagon $ABCDEF$ given the following data: $AB = 2\frac{3}{4}$ in., $BC = 1\frac{1}{2}$ in., $CD = 1\frac{1}{4}$ in., $AF = 1\frac{3}{8}$ in., $FE = 2$ in. Angle $ABC = 120^\circ$, angle $BCD = 70^\circ$, angle $BAF = 130^\circ$, angle $AFE = 105^\circ$.

Sketch the figure roughly as in the previous example and insert the particulars supplied (Fig. 44A). Draw AB (Fig. 44B) to the given length, then work around to D on the one side and E on the other, using the protractor for obtaining the angles. ED , the closing line, locates itself.



EXERCISES

1. The sketch represents the corner of a tiled room. The smaller hexagons have 3 in. sides, and the margins around them are 1 in. wide. Draw a right-angled portion of the tiling to a scale of 3 in. to 1 ft. (Ref. Fig. 35A.)
2. The design of a marquetry floor is based on the outline shown. The regular octagons have sides of 1 ft. and other measurements are given. Draw the portion of the floor to a scale of $\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 36.)
3. The sides AB and CD of the octagonal bay-window are half the lengths of the other sides. Set out the bay to the measurements provided. Scale $\frac{1}{4}$ in. to 1 ft. (Ref. Fig. 38.)
4. An octagonal chimney shaft measures 9 ft. across the parallel sides. Draw the outline of the section to a scale of $\frac{1}{4}$ in. to 1 ft. (Ref. Fig. 38.)
5. Set out the hexagonal bay to a scale of $\frac{3}{8}$ in. to 1 ft. (Ref. Fig. 35.)
6. Draw the section of the chimney stack to the dimensions given. Scale $\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 37.)
7. Draw all regular polygons from five to ten sides on a single base $1\frac{1}{2}$ in. long. (Ref. Fig. 39.)
8. The head of the paling is in the form of a regular pentagon. Set out the paling full size. (Ref. Fig. 41.)
9. A piece of tin in the shape of a regular heptagon has to be cut from a strip $3\frac{1}{2}$ in. wide. Set out the largest heptagon obtainable, such that one side of the heptagon is perpendicular to the edges of the strip. (Ref. Fig. 42.)
10. The central tower of a hospital is based on the design shown. All the figures are regular polygons. Set out the outline to the dimensions given, which are assumed to be taken from the plans.
11. Draw the plan of the dog-legged brick one-quarter full size.
12. A piece of lead is cut to the shape shown in the sketch. Set this out to a scale of $1\frac{1}{2}$ in. to 1 ft.
13. Draw the elevation of the keystone to the dimensions given.
14. An irregular plot of ground ABCDE has the following dimensions: AB = 250 yds., BC = $137\frac{1}{2}$ yds., CD = 200 yds., DE = 125 yds., EA = $87\frac{1}{2}$ yds., AC = $312\frac{1}{2}$ yds., BE = $262\frac{1}{2}$ yds. Draw the plot to a scale of 1 in. to 100 yds. What is the distance from E to C? (Ref. Fig. 43.)
15. Set out an irregular heptagon ABCDEFG to the following measurements: AB = 1.2 in., BC = 1.35 in., CD = 1.3 in., DE = 1.5 in., EF = 1.25 in., FG = .9 in., GA = 1.3 in., BG = 2.3 in., BF = 2.6 in., BE = 2.35 in., BD = 1.9 in. What is the length of GD? (Ref. Fig. 43.)
16. On a drawing the length LM measures $2\frac{3}{8}$ in. Complete the drawing to the same scale, and tabulate the distances LN and LO. The drawing is not to scale. (Ref. Fig. 44.)
17. Six similar isosceles triangles the equal sides of which are $1\frac{1}{2}$ in., and vertex angles 120° , are assembled together to form a regular hexagonal inlay. How would this be done and what would be the length between two parallel sides of the hexagon?
18. The centre pattern of a ceiling is in the form of a regular octagon. The octagon consists of eight isosceles triangles the equal sides of which are 6 ft. in length. Draw the pattern to a scale of $\frac{3}{8}$ in. to 1 ft.

CHAPTER IV

Circles, Arcs and Lines

This chapter is of the greatest importance to the young building student, as he is frequently encountering curves in some form or other in his work.

Circles in Contact

The underlying principles connected with circles in contact with other circles, arcs, and straight lines must be thoroughly grasped and understood before the student can hope to solve many of the problems with which he is likely to be faced in his building work.

Before attempting to solve any building problems on the circle, try to grasp a few of the elementary principles upon which practical problems are founded.

A circular arc can touch a straight line or another circular arc at one point only.

In Fig. 45 the arc touches the line at A, therefore the centre from which the arc was struck must lie on the perpendicular from A; if it were not on this perpendicular, the arc would not *touch* the line, it would *intersect* it. Similarly, in Fig. 46 the point of contact A of the two arcs, must lie on the straight line OO_1 joining the centres from which the arcs were struck. In Fig. 45 CD is the *tangent* and OA the *normal* to the curve.

Fig. 47 shows two arcs of given radii R and r in contact. To draw these arcs geometrically, we should draw first arc AP and radius r continued indefinitely. With O as centre and radius $r + R$ strike a small arc to intersect r continued to O' . As OO' is a straight line, P is bound to be the point of contact between the arcs.

EXAMPLE. (Fig. 48.) *Two circles of centres O and O' are given, and we require a circle of radius R to touch the given circles.*

It should be obvious that the centre of the required circle will be R distant from both circumferences, or $R + R'$ distant from centre O, and $R + R''$ distant from centre O' . Therefore with centre O, radius $R + R'$, strike an arc to intersect another arc struck from centre O' with a radius equal to $R + R''$. This intersection O'' is the centre of the required circle, as it is R distant from both given circles.

Now apply these simple constructions to a few practical examples.

EXAMPLE. (Fig. 49.) *The bottom flange of a rolled steel joist (R.S.J.) is given. It is required to copy the figure.*

First set out all straight lines. (The mean thickness of the flange is taken midway between A and C (Fig. 49).). The diagram will now be similar to that shown on the right half of Fig. 49A. (Note.—The flanges are broken to show a more comprehensive view.) To obtain the arc of centre O draw parallel lines $\frac{1}{8}$ in. distant from the web and flange respectively: O is now equidistant from both lines. O' is obtained in a similar manner, i.e. draw lines parallel to and $\frac{1}{8}$ in. distant from DE and AD. O' is the centre of the arc. (Note:—In practice, the " O' " arc is smaller than the " O " arc.)

Fig. 50 is the outline of a corner panel of an ornamental ceiling. The arcs are all struck with r radius. Draw an inner parallel triangle r distant from the outer triangle. The corners of this inner triangle must be the centres of the arcs as they are each r distant from the two lines forming tangents to each of the arcs.

Fig. 51 is the head of a pick. The lengths of the radii of the various curves are given. As the total length L is also given, the two main curves of R and R' radii can be drawn first without difficulty. To obtain the centre of the " r " arc, first draw the " r " circle to touch the " R' " arc. Now centre O must be r' distant from both the other curves r and R' . With centre P , radius $R' - r'$ describe a small arc to intersect another arc struck from Q with a radius of $r + r'$.

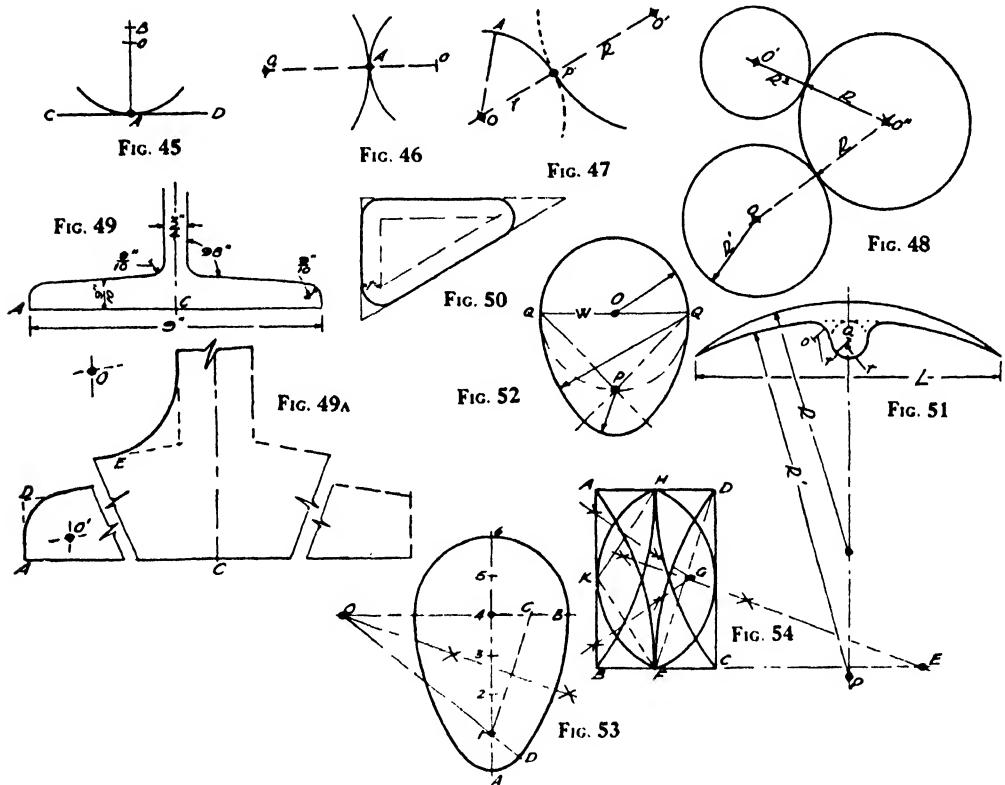


Fig. 52 shows the construction of circular arcs in contact relative to a w.c. seat. The width W is often 9 in. The construction is simple, all arcs being struck from centres O , P , and Q .

Fig. 53 represents the inner curve of an oval sewer. The total depth is usually given, in which case this distance is divided into six equal parts, the first (1) of which is the centre for the small arc at the base. Point 4 is the centre for the semicircular top. Centre O for the third arc must be equidistant from the other two arcs and on the line $B4$ continued, or it would not touch the semicircle at B , it would intersect it. Therefore make $BC = 1A$, join and bisect $C1$, and continue the bisector to O . O is the required centre as $OC + CB = O1 + 1D$.

Fig. 54 is a design for a leaded light, the length and breadth being given. Join HK , KF , and bisect them for point G the centre of arc HKF . Bisect DF for centre E . E must

lie on BC continued in order that arc DF may *touch* arc AF at F. This construction shows us that to find a point equidistant from three other points (in this example HKF) we must bisect two of the intervening spaces between the three points.

In Fig. 55 ABC are three points at unequal distances apart. O is equidistant from A, B and C, as by bisecting AC and BA, and joining B, A, and C to O, we have described two isosceles triangles OBA and OAC. But as $BO = AO$, $CO = AO$ as the triangles are isosceles. Therefore O is equidistant from A, B and C, or the centre of an arc or circle which would

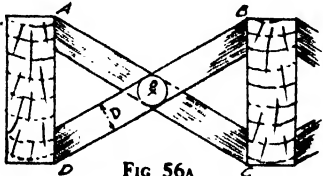


FIG. 56A



FIG. 55



FIG. 57

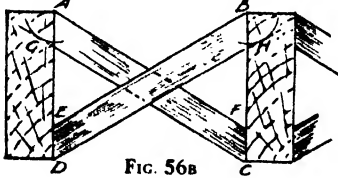


FIG. 56B



FIG. 62



FIG. 58

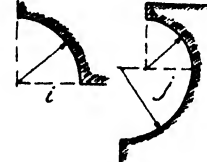


FIG. 59

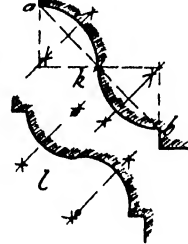


FIG. 60

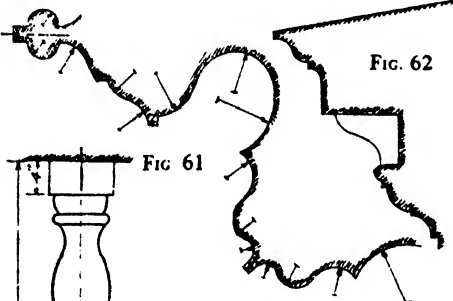


FIG. 61

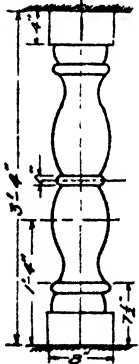


FIG. 63

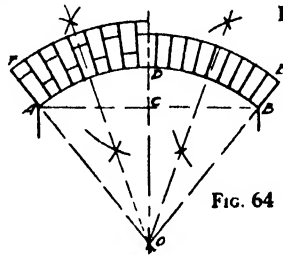


FIG. 64

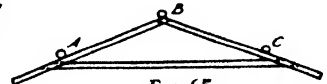


FIG. 65

pass through A, B and C. This construction is useful in the construction of a segmental arch given the span and the rise (Fig. 64).

The drawing of herring-bone strutting in a floor provides a simple but interesting problem on tangents to a circle.

EXAMPLE. (Figs. 56A and 56B.) To show 2" \times 1" herring-bone strutting between 9" \times 3" joists.

Method 1. Find the mid-point O in the rectangle ABCD. With O as centre describe a circle of 2 in. dia. Now draw tangents to this circle from ABCD.

Method 2. At A and B describe arcs of 2 in. dia. From C and D draw tangents to these arcs. Draw AF and BE parallel to CG and DH respectively.

Circles and Arcs Applied to Mouldings

Roman mouldings are made up chiefly of circular arcs. The following (Figs. 57 to 59) are some of the common types of Roman mouldings. The centres are distinctly shown and little difficulty will arise in the construction.

The names of the mouldings are as follows : Fig. 57 (*a*) Quirked bead, (*b*) Double quirked bead, (*c*) Staff bead, (*d*) Cocked bead, (*e*) Reed, (*f*) Flute. Fig. 58 (*g*) Ovolo, (*h*) Torus, (*i*) Cavetto or Cove, (*j*) Scotia, (*k*) Cyma Recta or Ogee, (*l*) Cyma Reversa or Reverse Ogee.

With the exception of (*c*) all the curves in Fig. 57 are semicircles.

In Fig. 58 (*g*) the ovolo centre may be at any point on the bisector of the chord. The true quadrant gives as good a profile as any, but this is not always possible, as the width of the moulding is often different from the depth.

In practice the torus (*h*) is often less than a semicircle.

The cavetto shown (*i*) is a quarter-circle, but like the ovolo the centre may vary according to the width and depth of the moulding.

Two quadrants form the scotia (*j*). The centres are on the same horizontal line.

The curved parts of the mouldings in (*k*) and (*l*) may be "flat," i.e. struck with a fairly long radius, or they may be quarter-circles as shown in (*k*). The line *ab* is first bisected, then each portion is again bisected : the centres of the arcs will lie on these bisectors.

Fig. 59 shows the effect of taking short and long radii for the cyma recta.

Compound mouldings are built up of two or more simple mouldings (Fig. 60 and Fig. 61). Two simple mouldings compose the section of the plinth Fig. 60, and Fig. 61 represents the section of a moulded column sometimes seen in churches. This compound moulding is formed entirely of circular arcs and straight lines.

When designing a compound moulding for any purpose, you will be well advised to build up the complete moulding with a number of simple recognized mouldings of the types illustrated in the preceding figures. Further, however small the scale may be to which the moulding has to be drawn, all members of the moulding should be clearly and distinctly shown, and not drawn in a slipshod manner because of their smallness.

Fig. 62 is a section through a stone block cornice ; all the mouldings employed are simple arcs and straight lines.

Fig. 63 shows the application of arcs in contact, to a stone baluster.

Arcs applied to Arches

In the following examples circular arcs are applied to various common arches.

EXAMPLE. (Fig. 64.) *To draw a segmental gauged arch given the span AB and the rise CD.*

Allow the bisector of an imaginary chord AD to intersect DC continued to O : this is the centre for striking the arch. The skewbacks at A and B, and all the voussoirs in the arch radiate to O. Draw the upper arc or *extrados* by making BE equal to the depth of the arch on the face. Set out on the extrados widths equal to the thickness of the voussoirs employed (say 3 in.). The number of voussoirs, of course, depends on the span of the arch, but in all cases the extrados should be divided in such a manner that an *odd* number of voussoirs will form the arch. In actual practice, the arc to pass through three points is often struck by moving a triangular frame made of thin battens, around and inside the space occupied by the points ABC (Fig. 65). By keeping the sides of the frame in contact with C and A respec-

tively, the apex of the frame will trace an arc passing through ABC. This construction is based on the theorem that all angles in the same segment are equal (Fig. 13). This is a reversal of that theorem, i.e. similar angles subtending the same chord must lie in the same segment.

In a rough arch, the mortar joints are wedge shaped, and in consequence the construction is somewhat different to that of the gauged arch.

EXAMPLE. (Fig. 66.) *To draw a segmental rough arch of two half-brick rings, given the span and the rise.*

For the outline of the arch, proceed as in the last example (making $HF = 9$ in.), with the extra arc CD midway between GH and EF. Now draw the four extreme voussoirs 3 in. wide at HDF and GCE. At the point O describe a small circle 3 in. dia., and divide the arcs GH and CD into 3-in. units with the dividers or compasses. From all these points draw tangents to the 3-in. circle. (Only that part of each tangent intersecting the arch need be shown.) A reference to Fig. 66A will illustrate clearly the reason all the voussoirs appear to be rectangles. By drawing double tangents to a 3 in. dia. circle to pass through two points 3 in. apart, a rectangle is formed, and as $HD = DF = 4\frac{1}{2}$ in., the rectangle will be $4\frac{1}{2}'' \times 3''$, or the size of the voussoir end exposed to view.

Fig. 67 is a semicircular rough brick arch of two half-brick rings. The voussoirs are drawn in a similar manner to those of the last example.

The only other common brick arch which need be included in this chapter is the flat or camber arch. This arch is called a camber arch on account of its intrados having a slight curve or camber; on a drawing the intrados is usually shown as a straight line. Two examples of camber arches are shown in Fig. 68.

EXAMPLE. (Fig. 68.) *To set out a gauged brick camber arch of given span and depth of face.*

The right half of Fig. 68 is an arch with a face depth of 14 in., the left half has a 9 in. face.

The apex of the equilateral triangle ABO is the radiating point for the skewbacks BE and AC, and the voussoirs. Dealing with the left half first, describe the arc CD with centre O, and divide this arc into 3-in. units (or whatever size the voussoirs are at their greatest thickness). From each of these units draw radiating lines to O; the parts of the lines crossing the face of the arch will be the joint lines required. You will perhaps wonder why we do not step off the 3-in. units on the extrados CJ. The reason is that were we to do so, every voussoir would require a different templet for cutting. With the method described, one templet can be made to cut every voussoir, because the arch can be contained in a part of the two concentric arcs *cdgh* (Fig. 68A). The templet would be cut to the shape *lmno* and adjusted to the required position on the voussoir. Do not assume that all the voussoirs are alike, they are not; but they can all be contained in the templet *lmno*.

The construction for the right half of the figure is similar to that for the left half, all voussoir joints radiating from the arc FE.

Nowadays gauged brick arches are often made completely in the builders' yards, and simply placed in their correct positions in the building.

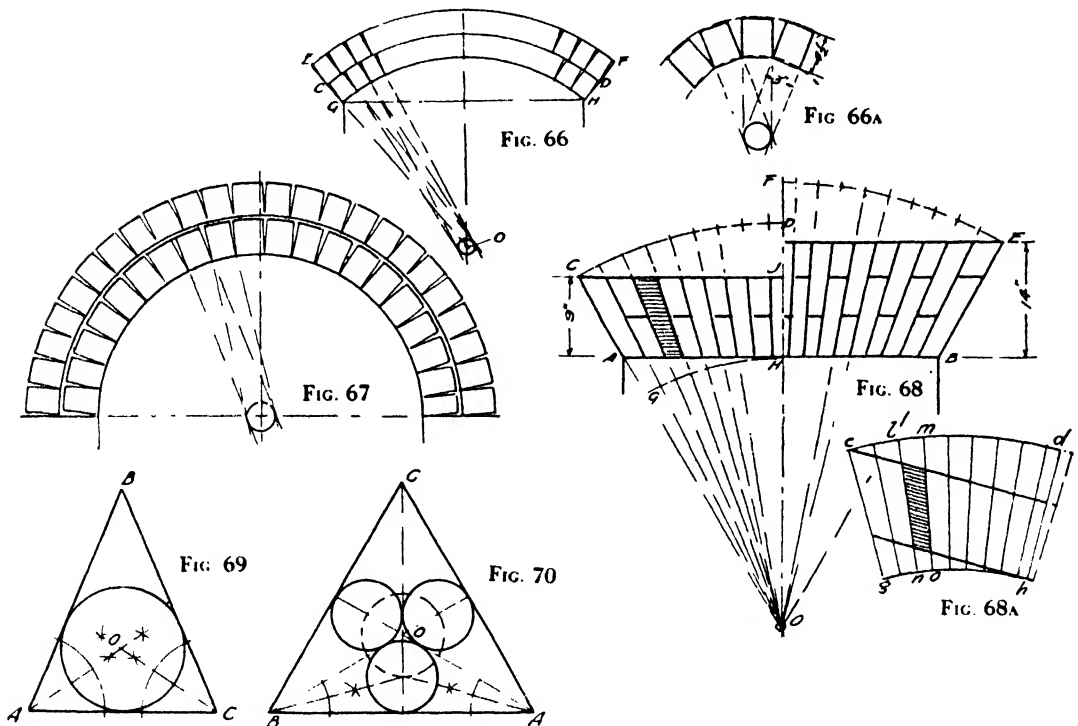
No difficulty should arise in drawing the common stone arches, as their geometrical constructions are similar to those for the brick arches.

Other arches are included in the chapter on the ellipse.

Inscribed Circles and Arcs Applied to Tracery

There has been no attempt at elaboration in the following examples. The geometrical principles underlying tracery designs have been clearly illustrated and described, but only briefly, as the average building student—particularly the trade student—may never be required to either design or make a tracery window. On the other hand, the geometrical principles involved are useful, and may be applied to other branches of building. In one example only (Figs. 72A, B and c), has any attempt been made to “build up” the complete design.

We will commence with a simple problem which will act as a basis for many of the designs that follow.



EXAMPLE. (Fig. 69.) *To inscribe a circle in any triangle ABC.*

Bisect any two angles. The intersection O of the bisectors is the centre of the required circle.

From this example many others may be evolved (Figs. 70 to 72).

EXAMPLE. (Fig. 70.) *To inscribe three equal circles in an equilateral triangle ABC, each circle to touch two other circles and one side of the triangle.*

Divide the triangle into three equal isosceles triangles ABO, etc. Now inscribe a circle in triangle ABO by bisecting any two angles. With compass point on O describe a fine circle passing through the centre of the circle just obtained. The intersection of this fine circle with BO and AO continued will locate the centres of the other two circles.

EXAMPLE. (Fig. 71.) *To inscribe in a regular polygon as many circles as the figure has sides, each circle to touch two other circles and one side of the polygon.*

From the centre of the polygon draw lines to all the angles, thus forming as many isosceles triangles as the figure has sides. Inscribe a circle in one of the triangles ABO, and reproduce a similar circle in the other triangles by the method shown in the last example.

In the following example, the inscribed circle to a triangle is used as a basis of design and elaborated to form a circular tracery window (Figs. 72A, B and C).

Fig. 72A is the simple outline of four circles inscribed in a square, the construction being similar to that in the preceding examples. Fig. 72B contains the same construction, but the circles are now incomplete and a fifth circle circumscribes the other four. Fig. 72C is the complete design. If you have drawn Fig. 72A accurately, Fig. 72C will require but little skill in drawing given suitable measurements.

A circle cannot be inscribed in *any* four-sided figure ABCD (Fig. 73) such that the circle touches all the sides. We can draw the circle to touch three of the sides but not always the fourth. In certain quadrilaterals, however, we can inscribe a circle truly, i.e. to touch all the sides. Such a quadrilateral must have all the bisectors of its angles intersecting at a point.

EXAMPLE. (Fig. 74.) *To inscribe a circle in a kite ABCD.*

The line CA bisects two of the internal angles of the kite, therefore the figure being symmetrical about CA must have the bisectors of the other two angles intersecting on CA at O; this is the centre of the required circle. Figs. 75 and 76 are inscribed circles based on the construction of Fig. 74.

EXAMPLE. (Fig. 75.) *To inscribe three equal circles in an equilateral triangle ABC, each circle to touch two other circles and two sides of the triangle.*

Draw the three medians of the triangle, i.e. lines from each angle to the mid-point of the opposite side. We have three kites, ADOE, BDOF, and CEOF. In one of these kites (say ADOE) inscribe a circle. Now locate the centres of the other two circles by means of the fine construction circle centre O. This construction may also be applied to any regular polygon as illustrated in Fig. 76.

Now consider a few examples of inscribed semicircles.

EXAMPLE. (Fig. 77.) *To inscribe a semicircle in an equilateral or isosceles triangle ABC.*

This and similar problems are based on the theorem that all angles contained in a semicircle are right angles. Draw the median CF, and make angles BFE and AFD each equal to 45° ; this means, of course, that angle DFE equals 90° . The arc struck from centre O must then be a semicircle as it contains the right angle EFD.

Fig. 78 shows this construction applied to a trefoil, i.e. an equilateral triangle containing three semicircles with adjacent diameters, each semicircle touching one side of the triangle.

Draw the medians of the triangle. In triangle ADB describe semicircle FEH by making $BHE = AHF = 45^\circ$. Construct the equilateral triangle EFG with EF as one side, and complete the trefoil.

Other inscribed semicircles are shown in Figs. 79 to 81.

EXAMPLE. (Fig. 80.) *To inscribe in a polygon as many semicircles as the figure has*

sides, each semicircle to touch one side of the polygon, and all the semicircles to have adjacent diameters.

Divide the polygon into the appropriate number of triangles from O the centre of the

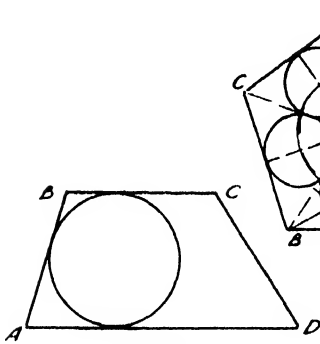


FIG. 73

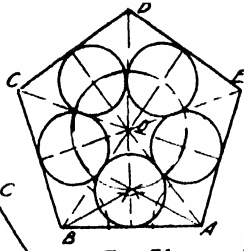


FIG. 71

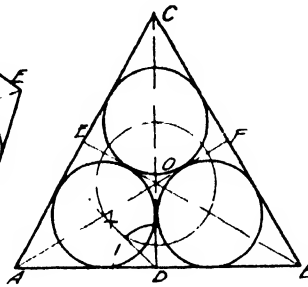


FIG. 75

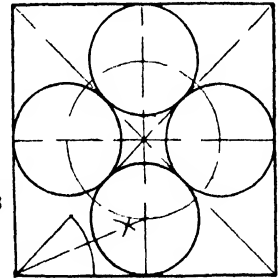


FIG. 72A

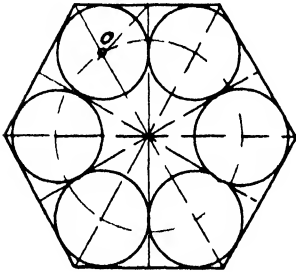


FIG. 76

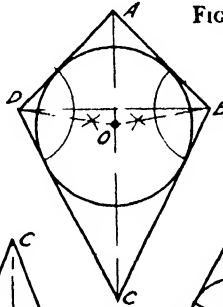


FIG. 74

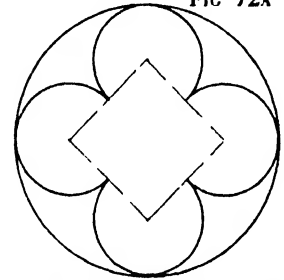


FIG. 72B

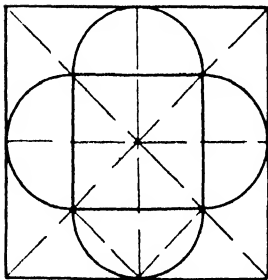


FIG. 79

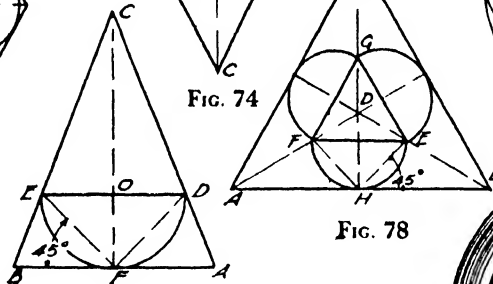


FIG. 78

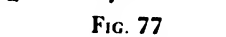


FIG. 77

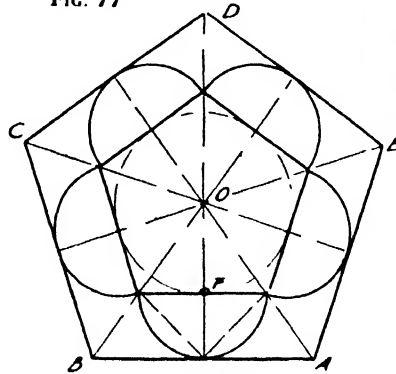


FIG. 80

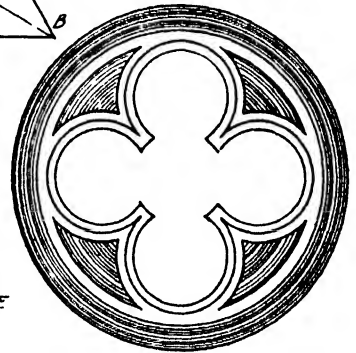


FIG. 72C

polygon. In triangle AOB construct the semicircle as in the last example. With centre O radius OF , describe a circle to locate the centres of the other semicircles.

Any number of equal semicircles can be inscribed in a circle by using a similar construction.

EXAMPLE. (Fig. 81.) *To inscribe six equal semicircles with adjacent diameters in a circle. Each semicircle to touch the circumference of the circle.*

Divide the circle into six equal sectors, and each sector again into two equal sectors from point O. Consider one of the sectors OCED. Draw AB perpendicular to OE, thus forming (with the tangent AB) the triangle OAB. Make angles AEG, BEF each equal to 45° . GF is the diameter of the first semicircle. Locate the other centres by means of the inner circle centre O.

Fig. 79 is a quatrefoil, the construction of which is similar to that in Fig. 78.

To inscribe semicircles in plane figures such that the semicircles touch *two* sides of the figure involves quite a different construction.

EXAMPLE. (Fig. 82.) *To construct a trefoil such that each semicircle touches two sides of the triangle.*

Draw the medians of the triangle ABC, and parallel to median BG draw DE. Make $DF = DE$, and join C to F; this line intersects BG at H. Construct the equilateral triangle HKL. J is the centre of one semicircle. Complete the trefoil as shown.

EXAMPLE. (Fig. 83.) *To construct a quatrefoil, each semicircle to touch two sides of the square.*

The construction differs but little from that of the last example. Make $OF = OE$, join F to C intersecting OE at G, and complete the inner square. The centres of the semicircles are plainly shown.

EXAMPLE. (Fig. 84.) *To inscribe six equal semicircles (with adjacent diameters) in a hexagon, each semicircle to touch two sides of the hexagon.*

Draw all diagonals and diameters of the hexagon. Consider the trapezium OGEH. In this figure we have to inscribe the semicircle with VU as diameter. On GH as diameter construct a semicircle, and through its centre P, draw PQ perpendicular to EH. Connect O to Q to intersect DE at S. Draw ST parallel to QP. T is the centre of the semicircle inscribed in the trapezium OGEH. Complete the inner hexagon, thus obtaining the centres of the other semicircles.

EXERCISES

1. Draw two lines of indefinite length to intersect at an angle of 45° about their mid-points, thus forming two angles of 45° and two angles of 135° . In the smaller angles describe circles of $\frac{3}{4}$ in. radii, and in the larger angles circles of $1\frac{1}{4}$ in. radii, each circle touching the two lines. (Ref. Fig. 50.)

2. On a woodworking machine, three pulleys of dia. 6 in., 8 in., and 4 in. are in mutual contact. Draw the pulleys to a quarter-scale showing the construction for finding the centres clearly. (Ref. Fig. 48.)

3. The part plan of a pergola is given. The outer circle is 8 ft. dia., the inner circle 7 ft. 6 in., and the smallest circles (of which there are eight) have diameters of 6 in. The curves connecting the small circles are semicircles. Draw a complete plan of the pergola to a scale of $\frac{1}{4}$ in. to 1 ft.

4. Set out the window opening to a scale of $\frac{3}{4}$ in. to 1 ft. (Ref. Fig. 58, g.)

5. Part of a curtail step is given in the diagram, the centres of three of the circular arcs being at three corners of the square ABCD, and the fourth centre being at E; $AE = 1\frac{1}{2}$ in., and the square ABCD has $\frac{3}{4}$ -in. sides. Draw the outline of the step. (Ref. Fig. 58, j.)

6. The end view of a butt hinge in its closed position is shown. Draw the hinge when it is opened at an angle of 135° . Scale twice full size.

7. The design for leaded light consists of arcs of circles contained in a rectangle 2 ft. 3 in.

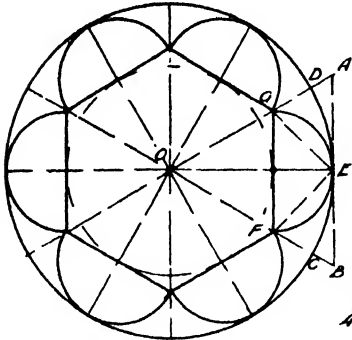


FIG. 81

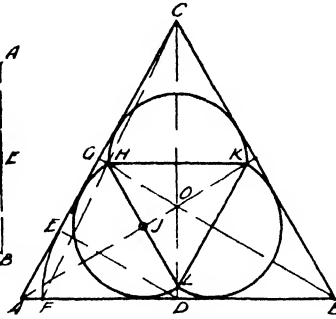


FIG. 82

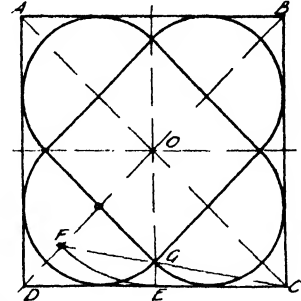


FIG. 83

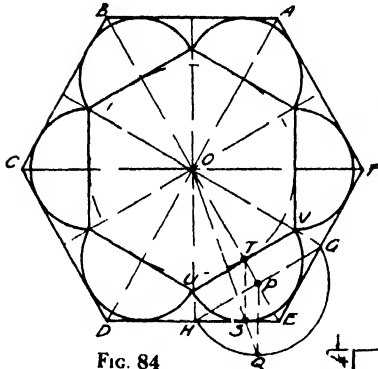
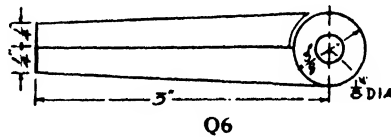
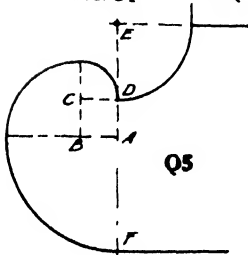
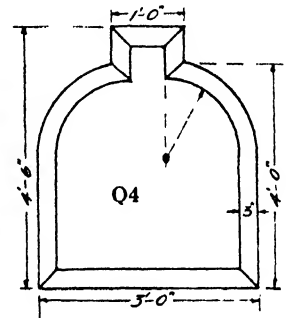
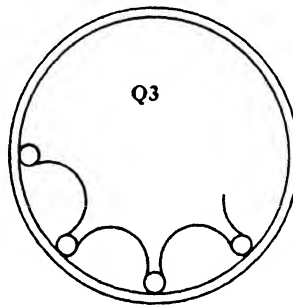
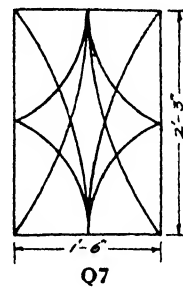


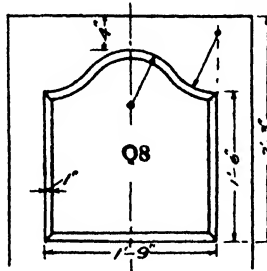
FIG. 84



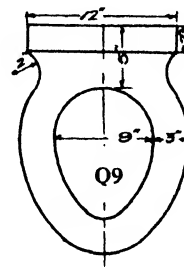
Q6



Q7



Q8



Q9

long and 1 ft. 6 in. wide as shown in the diagram. Draw the light to a scale of $1\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 54.)

8. Draw the upper panel of the door to a scale of $1\frac{1}{2}$ in. to 1 ft. The three arcs on the under edge of the top rail are struck with similar radii. (Ref. Fig. 59.)

9. Draw the w.c. seat to the dimensions given. Scale one-quarter full size. (Ref. Fig. 52.)

10. The pipe flange shown measures $4'' \times 2\frac{1}{2}''$ over all, the bore of the pipe being $1\frac{1}{2}$ in. : other dimensions are given. Draw the flange to a three-quarter scale. (Ref. Fig. 53.)

11. A floor consisting of $9'' \times 3''$ joists and $6'' \times 1\frac{1}{4}''$ tongued and grooved floor boards is stiffened by means of $2\frac{1}{2}'' \times 1''$ herring-bone strutting, the joists being placed 16 in. centre to centre. Draw a cross-section of about three joists with strutting and boards complete to a scale of $1\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 56.)

12. The depth of an oval sewer (inside dimensions) is 3 ft. 6 in. Draw the sewer to a scale of 1 in. to 1 ft. (Ref. Fig. 53.)

13. The brackets for supporting the upper portion of a pantry cupboard are shaped as shown in the diagram. Do not scrupulously copy the figure, but design a similar bracket using the dimensions given. Scale half full size.

14. The joint between the tread and riser in a wood stair is given. The tread is 1 in. thick, the riser is $\frac{3}{4}$ in., tongued into the tread $\frac{1}{4}$ in. The cove moulding is $\frac{5}{8}'' \times \frac{3}{4}''$. Draw the detail full size. (Ref. Fig. 58, *g* and *i*.)

15. A simple plaster cornice is shown in the diagram. The main arc is struck from centre A. The radii of B and C are $\frac{1}{2}$ in. Draw the cornice half full size. (Ref. Fig. 54.)

16. The diagram represents the part section through the style of a door showing a raised panel and stuck moulding on one side, and a bolection moulding on the other. Set out the section to a full-size scale, using your own discretion for the measurements not given.

17. The two main measurements of a moulded string-course are given on the sketch. Draw the section using your own judgment for the remaining measurements. Scale three-quarters full size. (Ref. Fig. 59.)

18. The moulded portion of a skirting-board is given. Draw the detail to a full-size scale. (Ref. Fig. 54.)

19. Draw the sash-bar full size to the dimensions shown. Show distinctly the centres for striking the ovolo mouldings. (Ref. Fig. 54.)

20. In a survey, it is desired to locate a point P which is equidistant from three other points A, B, and C. A is 150 yds. from B, and 225 yds. from C; B and C are 300 yds. apart. How far is P from the three points? (Ref. Fig. 55.)

21. The pattern in a fanlight is shown in the figure. The curves ABC and DEF are semicircles of $\frac{3}{4}$ in. radius. Complete the design to the measurements given. (Ref. Fig. 54.)

22. A weathered stone cornice 2 ft. deep, projects 18 in. from the face of the wall. Design a suitable moulding for the cornice. Scale 1 in. to 1 ft. (Ref. Fig. 62.)

23. The finial in the figure is built up of straight lines and circular arcs. The arc with centre A has a radius of 1 in., arcs with centres B and C each have a radius of 2 in. Set out the finial to a scale of quarter full size.

24. A segmental gauged arch has a span of 3 ft., a rise of 9 in., and a face depth of 9 in. Set out the arch to a scale of 1 in. to 1 ft. (Ref. Fig. 64.)

25. A part elevation of the opening for a bull's-eye light is shown. The keystones are 1 ft. 6 in. deep on the face. Draw the *complete* elevation showing all the joint lines of the brick voussoirs between the four keystones. Scale $\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 64.)

26. Draw the front elevation of a gauged brick camber arch of 4 ft. span and 14 in. depth. Scale 1 in. to 1 ft. Draw separately the shape of the piece of templet material required from which all the voussoirs can be cut. (Ref. Fig. 68.)

27. Set out the stepped stone arch to a scale of $\frac{1}{2}$ in. to 1 ft. The span *ag* is 6 ft. 6 in., and the joint lines *ab*, *cd*, etc., are all 1 ft. 6 in. in length.

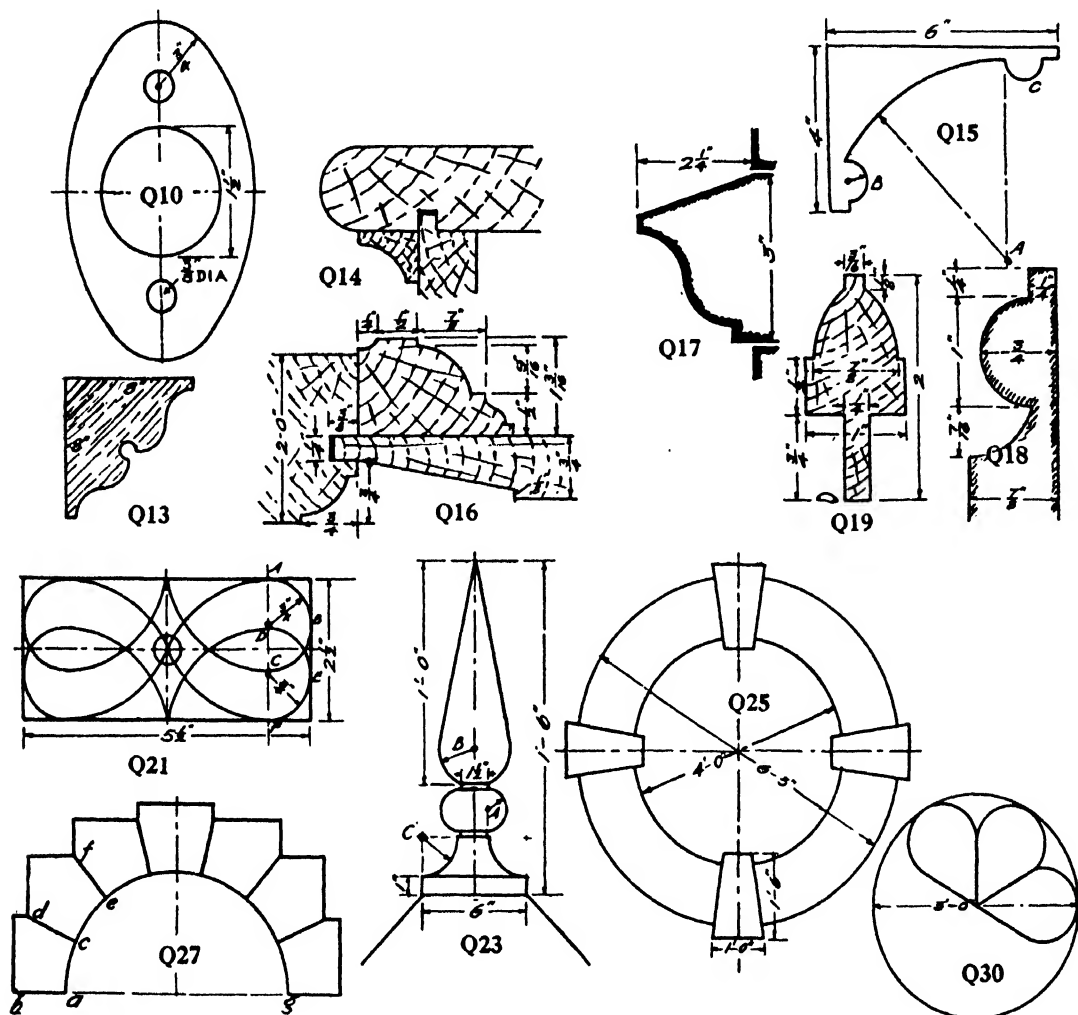
28. A rough brick arch (two half-brick rings) has a span of 3 ft. 6 in. and a rise of 1 ft. Set out the arch to a scale of $1\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 66.)

29. Inscribe three equal circles mutually in contact, in an equilateral triangle of 4 in. side, each circle touching one side of the triangle. Now elaborate this outline on similar

lines to that shown in Fig. 72 so as to form the design for a tracery window. (Ref. Fig. 70.)

30. The design for a circular light is based on the outline given. Set out the outline to a scale of 1 in. to 1 ft.

31. Draw a pentagon of 2 in. side, and inscribe in it five equal circles, each circle to touch two sides of the figure and two other circles. (Ref. Fig. 76.)



32. Draw a similar pentagon to that in the previous exercise, and inscribe in it five equal semicircles, having adjacent diameters; each semicircle must touch one side of the pentagon. (Ref. Fig. 80.)

33. Construct two types of trefoils in a triangle of 4 in. side. (Ref. Figs. 78 and 82.)

34. In a heptagon of $1\frac{1}{2}$ in. side, inscribe seven equal semicircles with adjacent diameters; each semicircle must be in contact with two sides of the heptagon. (Ref. Fig. 84.)

CHAPTER V

The Ellipse

You have probably been taught a few facts concerning the ellipse that will bear a closer investigation before proceeding to actual building problems with this curve.

The ellipse may be treated as the projection of a circle, i.e. as a plane curve, or it may be regarded as the section of a cone.

Fig. 85 shows three views of a circle. A is a full-faced view or a view taken at right angles to the face. C is a true edge view and is, of course, a straight line. B is neither a true face nor a true edge view; it is a view taken when the plane of the circle is inclined to the view-point: this view is an ellipse. We may say, then, that any view of a circle other than the true face and edge views will appear to be an ellipse.

If we cut a cone with an inclined plane as shown in Fig. 86A, the section thus formed is an ellipse. Fig. 86B shows a cylinder cut in a similar manner; this also gives an ellipse for a section.

Ellipse as a Plane Curve

For our purpose it will be sufficient if we treat the ellipse as a projection of a circle, and not as a conic section at this stage.

A true elliptical curve cannot be drawn with the compasses, as no part of the outline (or periphery) of an ellipse is part of a circle. In building we often make use of a curve which is spoken of as elliptical, but we do this for expediency, and the resulting curve is almost indistinguishable from a true ellipse. Also, peculiar as it may seem, a curve drawn parallel to a true ellipse is not itself a true ellipse, as the properties of an ellipse which are correct in the first case are absent in the parallel curve.

Fig. 87 is an ellipse with major axis DC and minor axis AB. The two points F, F' are *foci*—two very important points. To obtain the foci, take CO as radius, and with compass point at A or B cut the major axis at F, F'; i.e. make $FB = OC$. To draw the normal PQ, and the tangent PL at any point P on the periphery, join PF, PF' and bisect the angle thus formed for the normal; the tangent PL is perpendicular to the normal PQ.

A masonry arch with comparatively few voussoirs is usually made a true ellipse, and the joints between the voussoirs true normals, but a brick arch with many voussoirs is generally an imitation or quasi-ellipse, so made in order that fewer templates for cutting the voussoirs are required; this will be explained fully later in the chapter.

Various Methods of Striking

A good method of drawing a true ellipse is by means of a *trammel*. In small drawings the trammel may take the form of a strip of paper. In large ellipses such as those required for arches and arch centres, the trammel may be a long lath. In both cases the method of striking the ellipse is the same. In Fig. 88 DC is the major and AB the minor axis of

the ellipse. Mark on the trammel two points a and c such that $Pa =$ half the minor axis, and Pc half the major. Now keeping these points a and c on the major and minor axes respectively, mark a point at P . The trammel is shown in one particular position in the figure. Move the trammel around until the outline of a complete ellipse is formed, then trace a fair curve through the points marked at P .

Another mechanical method of drawing the ellipse is the "pin and cotton" method. If large ellipses have to be set out on the setting-out table or on the workshop floor, string and nails (or bradawls) are substituted for the cotton and pins. This is one of the best methods of drawing the ellipse, as the figure is theoretically true and no freehand drawing is needed. Briefly the method is as follows: Draw the major and minor axes and the foci. Insert pins at C , B and A (Fig. 89) and fasten the cotton around these three pins by making a loop on one end of the cotton and placing it on pin A , taking the cotton around pin B to pin C where it is wound a little, and anchoring the end of the cotton anywhere outside the

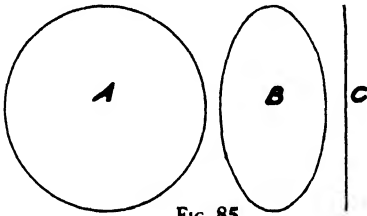


FIG. 85

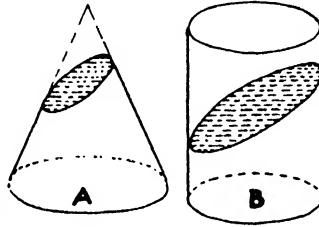


FIG. 86

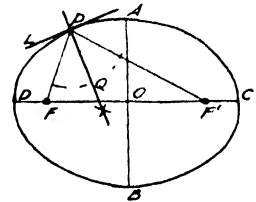


FIG. 87

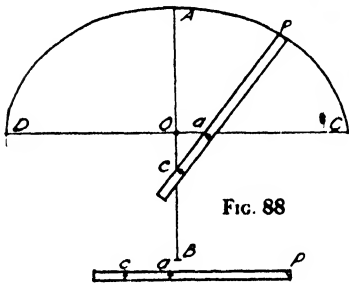


FIG. 88

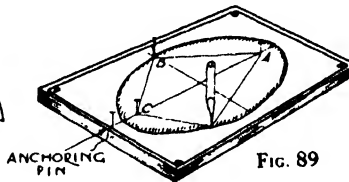


FIG. 89

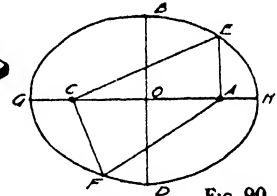


FIG. 90

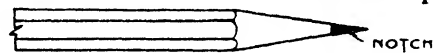


FIG. 91

required space for the figure. This end of the cotton can be held tightly with the finger or wrapped around a drawing pin. Now withdraw pin B and place the pencil point in the position previously occupied by B . The drawing is now purely mechanical. By allowing the pencil point to follow the cotton, always taking care that the cotton is taut, a perfect ellipse is described. Fig. 89 shows a second position of the cotton after the withdrawal of pin B with the pencil held upright describing the curve.

If the figure described in this manner is an ellipse, another important theorem is established. It is this: "That the sum of the distances from the two foci to any point on the periphery is constant." E.g. in Fig. 90, C and A are the foci, so $CE + EA = CF + FA = CB + BA$ and so on.

If a number of ellipses have to be drawn with the cotton and pins, a small notch near the pencil point will greatly facilitate the construction and prevent the pencil slipping away from the cotton (Fig. 91).

You will no doubt be acquainted with other methods of drawing the ellipse, so only a brief description will be necessary.

EXAMPLE. (Fig. 92.) *To draw an ellipse in a given rectangle.*

Divide OC and CE into the same number of equal parts (say four). Draw radiating lines from B to OC and from A to EC to intersect one another at the points indicated. Trace a fair curve through the points, and repeat the construction in the other quarters of the rectangle.

EXAMPLE. (Fig. 93.) *The ellipse projected from the circle, the major axis CD and the minor axis AB given.*

Draw two circles with major and minor axes as diameters. Divide the circles into any convenient number of sectors (say twelve) and draw lines parallel to the axes from the points where the dividing radii intersect the two circles. E.g. a line from F parallel to DC intersects a line from E parallel to AB. A number of points through which a freehand curve can be drawn are thus located.

Masonry Arches

EXAMPLE. (Fig. 94.) *To set out a parallel masonry arch of seven voussoirs given the span AB, rise OC, and depth of face AG.*

Locate the foci, and by one of the methods described in the previous examples, draw the semi-ellipse ACB. Divide this semi-ellipse (the intrados of the arch) into seven equal parts at DH . . . , connect these points to the foci and bisect each angle thus formed. The bisectors continued outwards represent the joints between the voussoirs, as they are true normals to the curve. Make DE, HJ, etc., equal to AG and complete the outer curve or extrados of the arch. The keystone is often allowed extra depth as shown.

Fig. 95 shows a stepped masonry arch. The construction is similar to that in Fig. 94 except in the formation of the extrados. A horizontal from E intersects a vertical from G at L, and this is repeated around the arch, thus forming "steps." The lengths of all the joints between the voussoirs are equal.

A practical method of striking an elliptical arch-mould in plaster is shown in Fig. 96.

A frame, in length nearly equal to the width of the opening, stayed by pieces F and G, is supported by the battens HJ, and wedged tightly between the jambs of the opening by wedges A and B. This frame which is made from light timber (about 3" \times 1") has a slot about $\frac{1}{2}$ in. wide left between each pair of battens to accommodate either wood pegs or bolts which are allowed to slide in the slots or grooves as they convey the trammel rod T around the arch. At C the plasterer fixes his zinc mould cut to the correct profile. This apparatus you will notice is a practical form of the trammel, so the pegs D and E must be the correct distance apart, i.e. the difference between the semi-major and semi-minor axes.

Approximate Ellipses

Many imitation ellipses so closely resemble true ellipses that they can hardly be distinguished from the latter. Further, a well-designed quasi-ellipse has advantages (in certain cases) over the true ellipse as will be shown.

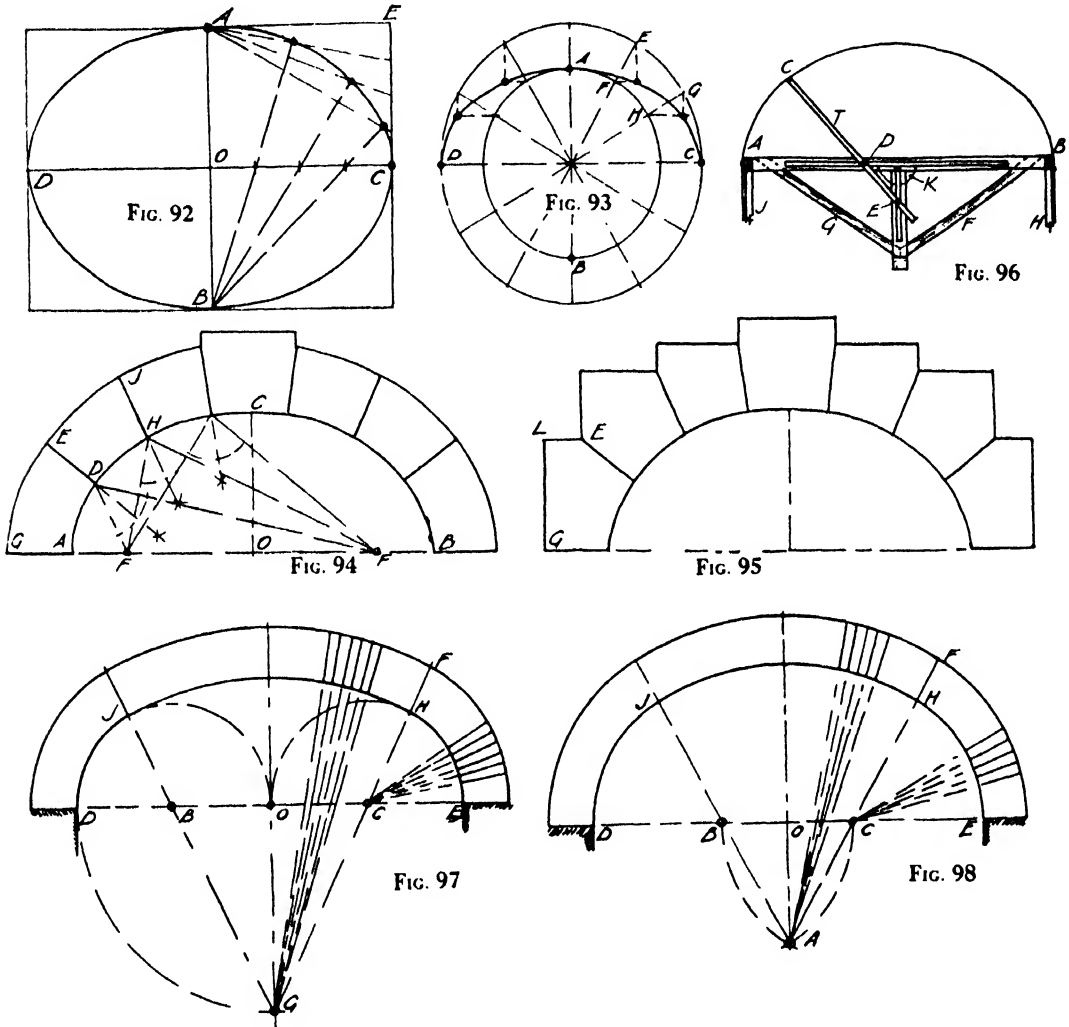
We will first draw two quasi-elliptical arches having given the span and *no rise*. These arches are called "three-centred" as they require three centres for striking.

Comparing Fig. 97 with Fig. 98, you will notice that for the same span, the arch in Fig. 98 has a greater rise.

In Fig. 97 the span DE is divided into four equal parts, and two semicircles described with centres B and C. OG is then made equal to OD. B, C and G are the three centres

required, so with G as centre, radius GH complete the intrados. As that part of the curve between H and E is struck from centre C, one templet or pattern will suffice for the voussoirs lying between these points, and another templet will be required for that part of the curve lying between H and J for the same reason, i.e. the curve HJ is one arc struck from centre G.

Fig. 98 is another form of three-centred arch with more "head-room" than that provided



in Fig. 97. The span is divided into three equal parts, and an equilateral triangle erected on the middle division BC. BCA are the three centres for striking the arch. In this case, the voussoirs between H and E require one templet, and those between H and J a different one.

Fig. 99 shows a very useful three-centred arch inasmuch as it can be made to accommodate any given span and rise. AB is the span and OG the rise.

Join GB, GA. Make $OF = OG$, and $GH = GJ = FB$. Bisect HB and JA with the bisectors intersecting at E. E is the centre for the arc MK, C and D the centres for the arcs MA and KB respectively. Joints of the voussoirs on arc KB radiate to D and those on arc KM radiate to E.

Fig. 99A is a five-centred arch struck from centres 1, 2, 3, 4, and 5.

The construction for striking is shown on the right, and a few voussoirs in position on the left of the centre line. This arch is perhaps as true an approximation to a real ellipse as it is possible to obtain.

To locate the centres, divide the semi-major axis OD and the line ED each into three equal parts. From the extremity of the minor axis (point B) draw radiating lines through the two points on the semi-major axis to intersect two lines drawn from A to the two points on ED; the intersection of these lines produces points *e* and *f*. Bisect Ae and allow the bisector to intersect the minor axis continued to point 1: this point is the centre for striking the arc Ae. Join e1, bisect *ef* and continue this bisector to intersect e1 at 2; this is the centre for striking arc *ef*. Connect *f*2. This line intersects the major axis at 3, and locates the centre for the arc *f*D. Voussoirs lying between *f* and D have their joint lines radiating to 3; those between *f* and *e* radiate to 2, and those between *e* and A radiate to 1.

The Ellipse in Common Mouldings

As Roman mouldings are usually built up from arcs of circles, Grecian mouldings are generally formed from other curves as the ellipse, parabola and hyperbola. Mouldings formed from the latter curves are considered by many to be more graceful than the Roman.

You will perhaps recognize Figs. 100A and B as types of cavetto or cove mouldings. These mouldings are quarter-ellipses which can be struck by any of the methods described earlier in the chapter.

The ogee and reverse ogee are shown in Figs. 101A and B. The two part ellipses forming the mouldings may be struck by any method. Those in the diagram are struck by the trammel and the radial line method respectively.

Fig. 102 is a torus mould. In this and the following example, the ellipse is formed inside a parallelogram. The method of construction, however, is similar to that of the ellipse inside the rectangle, and should easily be followed from the diagram.

The scotia in Fig. 103 requires little explanation, the diagram showing the construction fully.

An ovolo sash bar employing quarter-ellipses is shown in Fig. 104.

EXERCISES

1. Draw an elliptical masonry arch of nine voussoirs (including keystone) to the following dimensions: Span 12 ft., rise 4 ft., width on the face 1 ft. 3 in. The arch to have a parallel face. Scale $\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 94.)

2. Draw the arch of Exercise 1 with a stepped extrados. (Ref. Fig. 95.)

3. The part plan of an ornamental garden is given. All the paths are 4 ft. wide, and the three central beds are elliptical. Set out the complete plan to a scale of 1 in. to 10 ft., using your own discretion for measurements not marked.

4. A tunnel made of reinforced concrete 2 ft. thick is semi-elliptical in cross-section. The major axis (which is parallel to the ground surface) is 20 ft. and the semi-minor axis

is 8 ft. (inside dimensions). Draw a cross-section of the tunnel by the "projection of the circle" method. Scale 1 in. to 5 ft. (Ref. Fig. 93.)

5. A trammel rod for striking a bead around an elliptical arch is given. Set out the outline of the arch to a scale of $\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 88.)

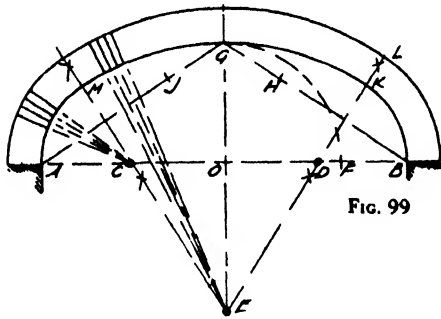


FIG. 99

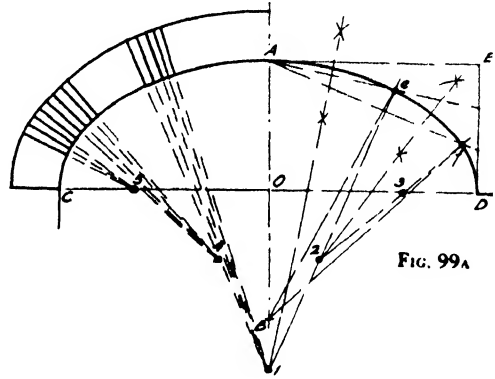


FIG. 99A

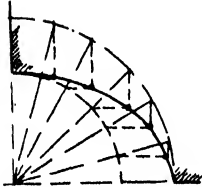


FIG. 100A

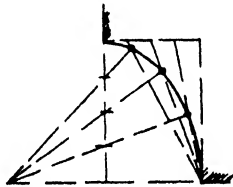


FIG. 100B

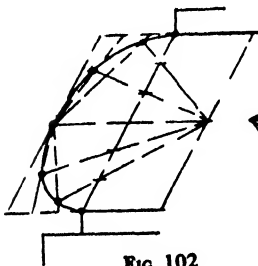


FIG. 102

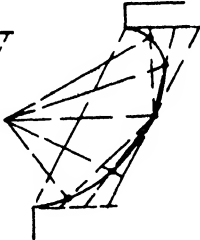


FIG. 103

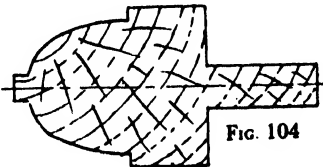


FIG. 104

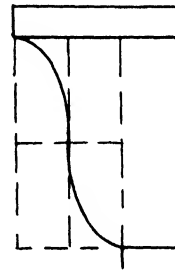


FIG. 101A

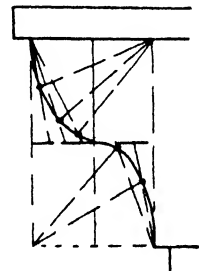
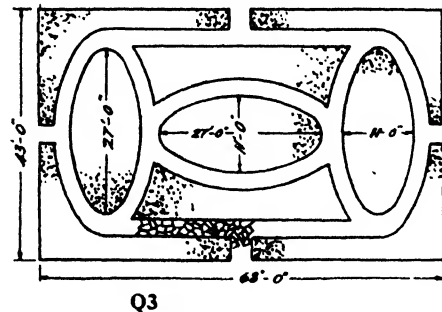
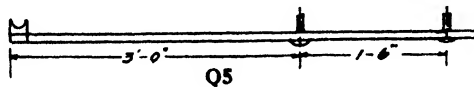


FIG. 101B



Q3



Q5

6. A small ornamental pond is approximately the shape of an ellipse. The total length of the pond to the outside of the 18-in. wall which surrounds it is 15 ft. The wall is surmounted by a stone coping 20 in. wide containing twenty stones. No width is given for the pond, but this is not vital, the only stipulation being that as few templets as possible are used for the coping. Set out the coping to a scale of $\frac{3}{8}$ in. to 1 ft. (Ref. Figs. 97 and 98.)

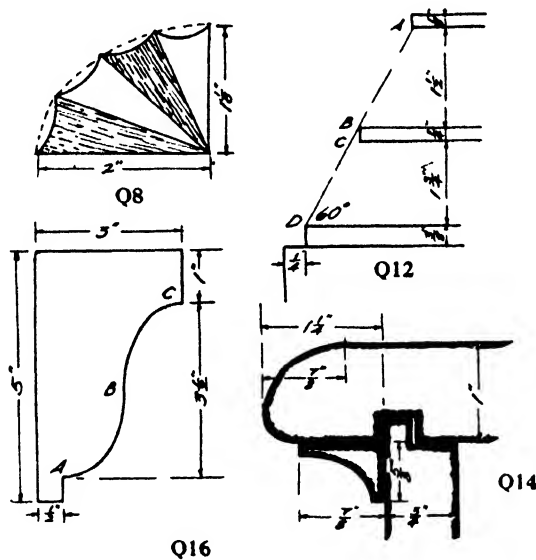
7. A gauged brick window opening roughly elliptical in shape, is 4 ft. wide and 1 ft. high from springing line. Set out the opening showing the voussoirs on the left side of the centre line. The voussoirs measure $9" \times 2\frac{1}{2}"$ on the face. Use a scale of $1\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 99.)

8. A quarter of an inlay is given. Complete the design.

9. A circular mirror of 2 ft. 6 in. dia. (without frame) when hung in position on the wall makes an angle of 40° with the wall face. Draw the plan and elevation of the hanging mirror to a scale of 1 in. to 1 ft.

10. Draw the elevation of the mirror in the previous exercise when it is surrounded with a 4 in. frame.

11. The major axis of the elliptical centre for a stone arch is 8 ft., and the foci are 6 ft. 6 in. apart. Set out the intrados of the arch to a scale of $\frac{1}{2}$ in. to 1 ft.



12. The figure given represents part of the return moulding for a skirting. The space between A and B is occupied by a Greek scotia, and that between C and D by a torus. Complete the moulding (Ref. Figs. 102, 103.)

13. The detail of the joint between a stair tread and riser is given with certain measurements omitted. Use the main measurements supplied, and design a similar detail, $1\frac{1}{2}$ times full size. (Refs. Figs. 102, 100A.)

14. A stone cornice 10 in. deep and overhanging 12 in. from the face of the wall is built up of Grecian mouldings and flat bands. The mouldings must be bold and not too small to be seen from the pavement below. Design a suitable cornice to a scale of 3 in. to 1 ft.

15. Design the bottom rail of a sash suitable for the window of which the bar in Fig. 104 is a unit. The bottom rail is $3\frac{1}{2}$ in. deep, 2 in. thick, and double weathered on its under surface.

16. The curves AB and BC in the given bracket are quarter-ellipses. Make a full-size drawing of the bracket, showing how you obtain the axes of the ellipses. (Ref. Fig. 101.)

17. The span of an elliptical arch is 5 ft., and the distance apart of the foci is 4 ft. 6 in. Draw the intrados of the arch to a scale of 1 in. to 1 ft., and show the direction of the joints between the voussoirs, assuming there are seven of the latter.

CHAPTER VI

Areas

In this chapter calculations have been eliminated as far as possible. It must be admitted that calculated answers are often more accurate than graphical answers, but this, to a great extent, is due to inaccuracy in draughtsmanship. On the other hand, many problems dealing with areas are more conveniently solved graphically, and, given a reasonable degree of accuracy in drawing, the solution obtained by drawing to scale often saves much time and labour which might be spent in calculations.

The apparatus used for measuring lines in the various branches of building will probably be familiar. For drawing, the scale rule is used; the workman uses the two-foot rule for small measurements and average work on the building; to measure small sites, the steel tape is convenient, and for larger tracts of land the surveyor uses the chain which is 22 yds. long and contains 100 links.

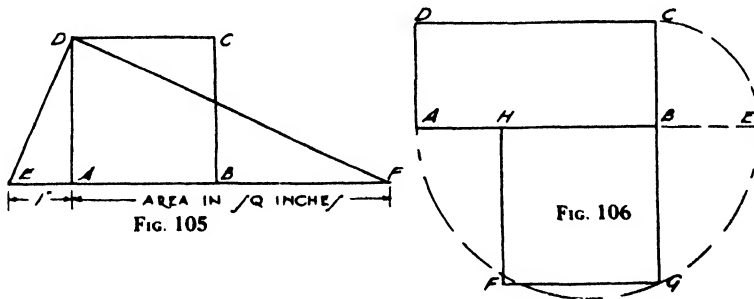
A true graphical solution to a problem on area is the length of a line representing the area of the figure; this involves no calculations whatever.

The usual method of ascertaining the area of any figure graphically, is to transform the figure to an equivalent square and then to proceed as in the following example.

Square, Triangle, Polygon

EXAMPLE. (Fig. 105.) *To find the area of the square ABCD in square inches.*

Make AE equal 1 in. Join E to D and D to F so that the angle EDF = 90°. AF measured in inches is the area of the square in square inches.



Test this by making $AB = 2$ in. Now as the area of the square is 4 sq in., the line AF should measure 4 in. If the area had been required in square centimetres, AE would have been 1 cm., and AF measured, of course, in centimetres.

EXAMPLE. (Fig. 106.) *A piece of lead ABCD is rectangular in shape. It is required to cut a square piece to have the same area as ABCD.*

Make $BE = BC$. Describe a semicircle on AE, and continue CB to intersect the semicircle at G. BG is the side of the required square. BG is called the *mean proportional* between AB and BC, i.e. a line which satisfies the proportion $AB : BG :: BG : BC$. Now

the product of the extremes equals the product of the means, so $AB \times BC = BG^2$, in other words, the area of the rectangle equals the area of the square.

EXAMPLE. (Fig. 107.) *It is required to make a square gutter to carry the same flow of water as the vee gutter ABC.*

The problem is to draw a square equal in area to the triangle ABC. As the area of a triangle equals $\frac{(\text{base} \times \text{height})}{2}$, by bisecting CK (the perpendicular height of the triangle)

at J and completing the rectangle ABED, we obtain a rectangle equal in area to the triangle. Now from the previous example (construction omitted) we transform the rectangle into a square FGIH (Fig. 107c) and add the 1-in. boards.

EXAMPLE. (Fig. 108.) *A pentagonal flower-bed ABCDE has to be re-designed in the form of a square without any alteration in area. To draw the square.*

This problem (and many other problems of a similar type) is based on the theorem that all triangles on the same base and between the same parallels are equal in area (Fig. 14). The first step is to draw a triangle equal in area to the pentagon. This is done by drawing EF parallel to DA and CG parallel to DB to intersect the base AB continued in both directions, and joining DF and DG. FDG is the required triangle.

Notice that the $\triangle DFA$ is equal in area to the $\triangle DEA$ because they are both on the same base (DA) and between the same pair of parallels (DA and EF). Therefore, by substituting $\triangle DFA$ for $\triangle DEA$, and $\triangle DGB$ for $\triangle DCB$, the area of the $\triangle FDG$ is the same as that of the pentagon. To complete the solution to the problem transform the $\triangle FDG$ into a rectangle, and the rectangle into a square. The required square is GHJK.

Fig. 109 shows the transformation of a hexagon ABCDEF into a triangle of equal area.

We can transform any polygon—regular or irregular—into a triangle by gradually reducing the number of sides without making any alteration in the area of the figure.

Commence this problem by joining A to C (or any other two alternate angles) drawing a parallel line through B to intersect AF continued to G, and joining GC. We have now substituted GC for CB and BA, without altering the original area, as the $\triangle AGC$ and the $\triangle ABC$ are on the same base AC and between the same parallels AC, BG. By repeating this construction at DEF we obtain a four-sided figure CDHG (reproduced for clearness). Join D to G, draw CK parallel to DG to intersect GH continued. The triangle KDH is now equal in area to the original irregular hexagon ABCDEF.

Field-book

In measuring an irregular plot of ground with straight boundary lines (Fig. 111) it is customary to set up sight rods at opposite corners of the plot and obtain an imaginary line. By means of an instrument for measuring or setting out angles (prismatic compass, or theodolite) points are obtained on this imaginary line such that the "offsets" HG, IB, etc., form right angles with the initial line AE. The offsets are measured and entered in columns in the Field Book, a page of which is shown in Fig. 110. From this survey, the plot can be drawn to scale and its area ascertained.

In the figure AH = 55 lks., AJ = 170 lks., JM = 50 lks., LF = 120 lks., and so on.

The areas of irregular curved figures are not satisfactorily ascertained graphically, and therefore will not be dealt with here.

Circle

A circle can be transformed into a square graphically without difficulty.

EXAMPLE. (Fig. 112.) *A square light has to be substituted for an existing circular light of 4 ft. dia. without alteration of area. What will be the length of the side of the square?*

Divide the radius OA graphically into seven equal parts. Draw the rectangle ABFO

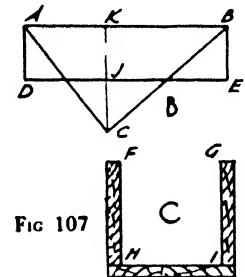
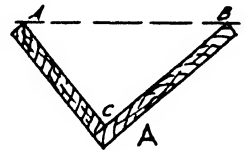
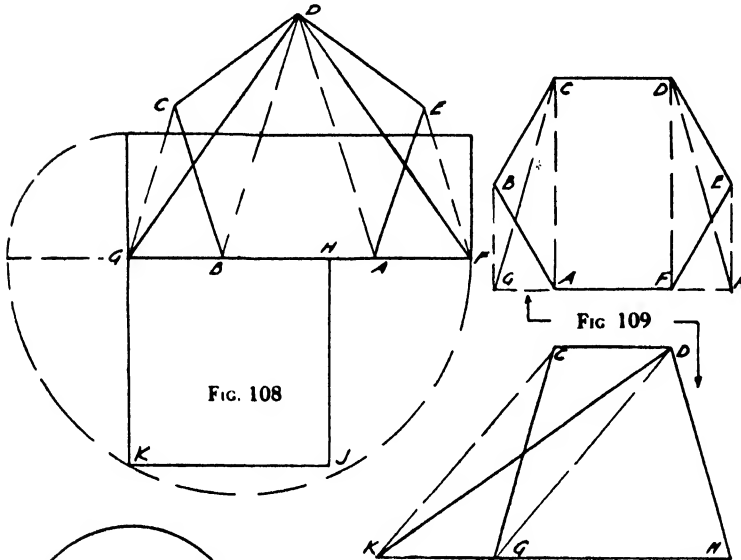
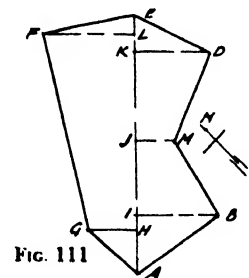
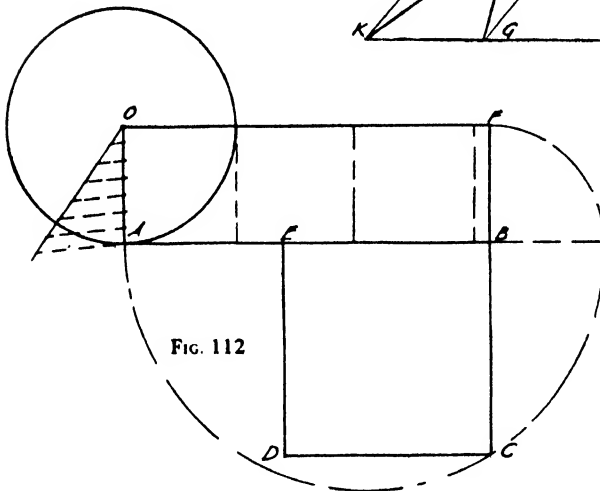


FIG. 110
Measurements in Links

120	28.2
250	30
170	50
72	108
63	38
A 160 N.E.	



equal in area to the circle by making $AB = 3\frac{1}{7}$ times AO . You will notice that this rectangle $= (AO)^2 \times 3\frac{1}{7}$. Now transform the rectangle into the square BCDE. BC is the required length.

Equivalent Areas

To transform a rectangle into another rectangle of different linear dimensions, but having the same area, consider the following Example.

EXAMPLE. (Fig. 113.) *The outline of a proposed house is a rectangle ABCD. For reasons of economy it is desired to alter this shape so that AE shall be the length of one side. No alteration in area is contemplated. To draw the revised outline.*

Draw ED, and parallel to this line FB. Construct the rectangle AEGF which will have the same area as the original rectangle ABCD.

This problem is instructive inasmuch that it illustrates that the nearer the shape approximates to a square, the less the length of the boundary line for the same area enclosed.

A rectangle $4'' \times 1''$ has an area of 4 sq. in. and a perimeter of 10 in. A square of 2 in. side has an area of 4 sq. in. and a perimeter of 8 in. This fact is important in the design of buildings, as not only is there a big difference in the length of walls, but foundation concrete, footings, plinths, skirtings, roof timbers, etc., will show a corresponding difference. So we might say, generally, that the square building is more economical to build than any other shape of building.

EXAMPLE. (Fig. 114.) *A trench ABCD is dug, and the excavated soil piled at the side of the trench in the form of an isosceles triangle of base BH. To draw the complete section of trench and earth.*

In practice, the earth would expand in bulk, and therefore the triangular section of earth would be greater than that shown in this solution.

Transform the quadrilateral ABCD into a triangle of equal area by joining CA, drawing DE parallel to CA, and joining CE. Reproduce this triangle at BFG. We have now to draw a triangle of the same area as BFG but having BH for a base. Draw HF and GJ parallel to it (the $\triangle BHJ$ is now equal in area to the $\triangle BFG$). Now draw the triangle BHK of the same area but isosceles, by making JK parallel to BH and K equidistant from B and H.

(If a 10 per cent increase in bulk had to be allowed for, the $\triangle BGF$ would be increased by adding one-tenth the length of the base on to BG, and using the same vertex F.)

Area Problems Based on the Right-Angled Triangle. Addition and Subtraction

Not only is the square on the hypotenuse of a right-angled triangle equal to the sum of the squares on the other two sides, but the area of any figure on the hypotenuse is equal to the sum of the areas of *similar* figures on the other two sides. E.g. in Fig. 115A the area on AB equals the area on BC plus the area on AC because all three figures are similar. In Fig. 115B the semicircle with AB as diameter equals the sum of the semicircles on AC and CB.

EXAMPLE. (Fig. 116.) *A water pipe of $1\frac{1}{2}$ in. dia. has to be replaced by two smaller pipes one of which must be 1 in. dia. What will be the size of the second pipe?*

Draw a right-angled triangle ABC making $AB = 1\frac{1}{2}$ in. and one other side 1 in. The remaining side BC will be the diameter of the second pipe, as the area of the combined circles constructed on AC and CB will be equal to the circle of diameter AB.

EXAMPLE. (Fig. 117B.) *It is required to construct some hexagonal tiles to have (a) twice the area, (b) half the area of a given tile ABCDEF. Ascertain the new sizes.*

We could answer this question by making AB (Fig. 117A) equal to one side of the given hexagon, constructing a right-angled triangle ABC, making $AC = AB$, and constructing a regular hexagon on CB; and for the second part of the question, making angle ADB a right angle, such that $AD = DB$, thus obtaining the side of the hexagon DB which would have half the area of the hexagon constructed on AB.

The better way of answering this and all similar problems is as follows :

On one side of the given hexagon AB construct two right-angled triangles, the first one ABG having AG equal to AB , and the second one ABH having AB for hypotenuse. Instead of constructing hexagons on sides BG and BH , describe arcs GI and HJ with B as centre. Draw radiating lines through the angles of the given hexagon from B . Commencing at J and I construct hexagons by drawing lines parallel to the given hexagon and terminated by the radiating lines as shown. We thus get a hexagon with IB as one side having double the area of the original hexagon, and a hexagon with side JB having half the area.

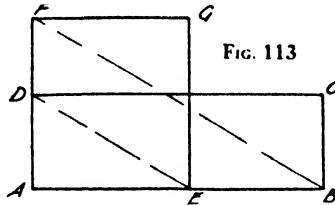


FIG. 113

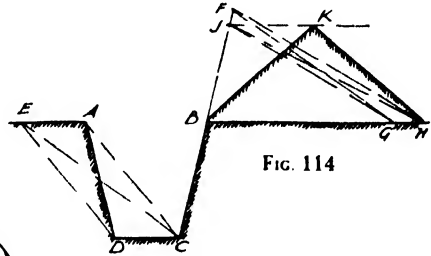


FIG. 114

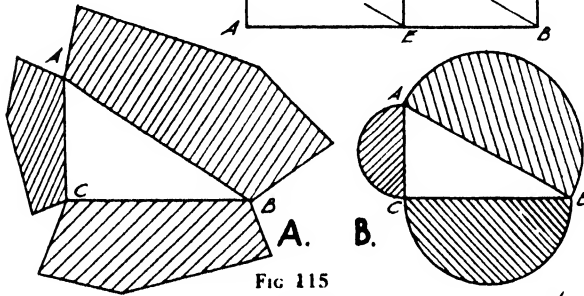


FIG. 115

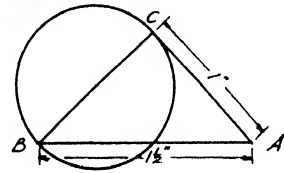


FIG. 116

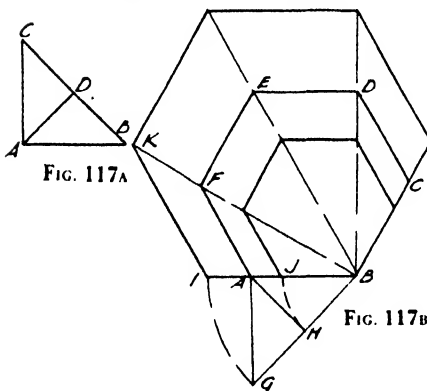


FIG. 117A

FIG. 117B

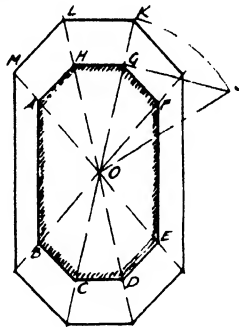


FIG. 118

This method is to be preferred because it eliminates the laborious copying of figures on the hypotenuse and other sides when the figures are not regular.

EXAMPLE. (Fig. 118.) *An octagonal lawn $ABCDEFGH$ has to have a path made round it, such that the path area equals the lawn area. To make a complete drawing of lawn and path.*

Locate the centre O of the octagon at the intersection of the diagonals. On one-half of one of the diagonals say OG , construct a right-angled triangle OGJ making $GJ = GO$. Make $OK = OJ$. Now OK is the length of the hypotenuse of a right-angled triangle, the sides of which are alike and equal to OG . From the point K construct an octagon parallel to given octagon. The triangle OKL is twice the area of triangle OGH , the triangle OLM is twice the area of triangle OHA and so on.

EXAMPLE. (Fig. 119.) *The plan of a plot of ground ABCDEF containing a pond GHIJK is given. To find the area of the ground.*

Reduce both figures to squares of equivalent areas respectively. This construction has already been explained (Fig. 109) and should not be difficult to follow. Substitute BL for BA and AF, CN for BL and BC, CM for CD and DE, thus obtaining a triangle NCM equal in area to the irregular hexagon. (This $\triangle CNM$ is reproduced for clearness.) Transform triangle CNM into the rectangle MN m , and the rectangle into the square NOPQ. Transform the outline of the pond into the triangle HRS, then to the rectangle RS sr , and finally to the square TUVR. We have now a square equal in area to the ground, and a square equal in area to the pond: we require a square equal in area to the difference of these two squares. Make NQ the hypotenuse of a right-angled triangle with side NW equal to side of small square RT. The square on the remaining side QW will equal the difference in area of the other two squares, i.e. the area of the ground less the area of the pond. Make W3 1 in., draw the right-angled triangle 3X2 and measure W2 in inches; this equals the required area in square inches. Now transform this answer to the units from which the plan was drawn, i.e. sq. mls., sq. yds., sq. ft., etc. If for example the scale used was 1 in. to 100 yds., 1 sq. in. would equal 10,000 sq. yds.; therefore if the line 2W measures 4 in., the area would be $(10,000 \times 4)$ sq. yds.

Division of Areas

EXAMPLE. (Fig. 120.) *Four houses are situated at vertex A of a triangular piece of ground ABC. Each house has to have an equal share of ground, and access to its own plot. To divide the land equally.*

Divide CB into four equal parts, and join each of these boundary points to the vertex A of the triangle. Each portion must be similar in area as all the bases CD, DE, etc., are equal, and the vertical height is constant for all triangles.

EXAMPLE. (Fig. 121.) *A pasture ABC triangular in shape has to be divided into four equal areas by fences fixed parallel to side CB. To draw the position of the fences.*

On an adjacent side to CB (say AC) describe a semicircle. Divide AC into four equal parts *fed* and from these points erect perpendiculars to AC to intersect the semicircle at *f'e'd'*. With centre A describe arcs *d'd₁*, *e'e₁*, and *f'f₁*. Draw parallel lines to CB from *d₁e₁f₁*. The triangle is now equally divided. In Fig. 122 a similar construction is adopted in dividing a polygon into five equal areas.

EXAMPLE. (Fig. 123.) *A triangular site ABC has to be equally shared between two persons who desire a party wall building perpendicular to side BC. To find the position of the wall.*

Locate E the mid-point of BC. From A draw AD perpendicular to BC. Now find the mean proportional BB₁ between BD and BE, i.e. make $BD_1 = BD$, describe a semicircle on D₁ E, and erect perpendicular BB₁. Make BF equal to BB₁ and draw perpendicular FG which is the position of the required party wall.

EXAMPLE. (Fig. 124.) *ABC is the plan of a field roughly triangular in shape: there is a pond at D. The field has to be fenced from D to side BC so as to divide the field equally into two parts with pond accessible to both.*

Transform the triangle ABC into the triangle BDE by joining DC, drawing AE parallel to DC and joining DE (\triangle s DCE and DCA are equal in area because they are on same base

EXAMPLE. (Fig. 125.) *To divide a site ABCD into two equal areas by a straight wall one end of which is at B.*

Draw AC and bisect it at E. BE, ED, would bisect the site as $\triangle ABC$ and $\triangle ACD$ are both bisected: but we require one straight line from B in place of BE, ED. Join B to D and through E draw EF parallel to BD. FB is the required wall.

(Note.— $\triangle BED = \triangle BFD$ as the base BD is common to both and EF is parallel to BD. But as BE and ED originally bisected the figure, then BF also bisects it.)

EXAMPLE. (Fig. 126.) *A fence BD is already in existence in a triangular plot ABC. Another fence has to be erected from D to side AC, such that the plot will be bisected.*

From F the mid-point of AC draw FB. Draw DF, and parallel to it BE; DE is the plan of the second fence.

(Note.—The \triangle s BEF, BED are equal in area being on the same base and between the same parallels. Therefore \triangle s BAE + BED = \triangle s BAE + BEF. But $\triangle BAF$ is half the area of $\triangle ABC$ \therefore quadrilateral BAED = half the area also.)

EXERCISES

1. A square room has 15 ft. 6 in. sides. Draw the room to a scale of $\frac{1}{4}$ in. to 1 ft., and find its area graphically. (Hint: If $\frac{1}{4}$ in. represents 1 ft., 1 in. will represent 4 ft., and 1 sq. in. will represent 16 sq. ft.) (Ref. Fig. 105.)

2. A door measures 6' 6" \times 2' 6". Without calculations find its surface area. (Ref. Fig. 106.)

3. A symmetrical gable end of a building is 25 ft. wide, 20 ft. from ground to eaves, and 30 ft. from ground to ridge. Find the surface area without calculations. (Ref. Fig. 107.)

4. Find the area of a hexagonal tile of 3 in. side. Graphical methods only must be employed. (Ref. Fig. 109.)

5. A page from a field book is similar to that shown, the measurements being in links. Draw the plot to a scale of 1 in. to 50 links and find the area in square yards. (Ref. Fig. 111.)

6. Ascertain graphically the area of a circular swimming bath of 25 yds. diameter. (Ref. Fig. 112.)

7. The measurements of a small plot of ground are tabulated as shown, the measurements being in yards. Draw the plot to a scale of 1 in. to 10 yds., find the area graphically, then check your result with a calculated answer. (Ref. Fig. 111.)

8. A rectangular piece of lead 1' 9" \times 1' 1" has been cut by mistake, the longer side being 3 in. in excess of the specified measurement. The weight of the piece is correct. Draw the correct shape cut from a similar thickness of lead to a scale of 1 in. to 1 ft. (Ref. Fig. 113.)

9. Two branch pipes of equal bore join a main pipe of 2 in. bore. What will be the bores of the branch pipes assuming their bores together equal the bore of the main pipe? (Ref. Fig. 116.)

10. Find graphically the area of the floor given. Compare your answer with a calculated answer. How many "squares" of flooring will be required to cover the area allowing 5 per cent for waste? (A "square" = 100 sq. ft.)

11. Two octagonal ventilating shafts of sides 6 in. and 8 in. respectively, have to be replaced by one octagonal shaft which must have the same effective cross-section area as the existing shafts. Set out the shape of the new shaft and measure the distance across its parallel sides. (Ref. Fig. 115.)

12. Design a room similar to that shown in Question 10 but having the side AB 15 ft. long instead of 21 ft. as given. (*Hint*: Draw radiating lines from B through all corners.)

13. The path around a circular lily pond of 12 ft. dia. is exactly equal in area to the water surface. What is the width of the path? (Ref. Fig. 116.)

14. The plan of a building with an enclosed area (marked A) is given. Find graphically the effective floor area, i.e. the total area of the plan less the area A. (Ref. Fig. 119.)

15. Find graphically the area of the floor given. How many squares of floor boards will be required allowing 5 per cent for waste?

16. The earth taken from a rectangular trench 4 ft. deep and 2 ft. 6 in. wide expands

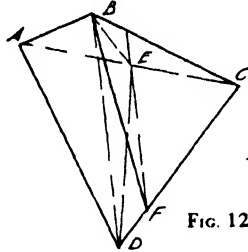


FIG. 125

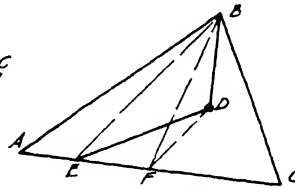
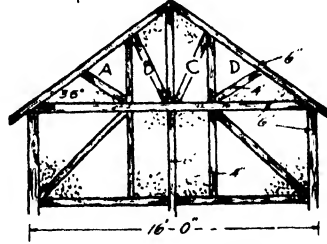
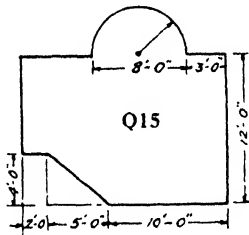
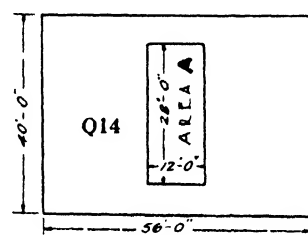
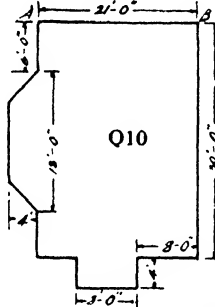


FIG. 126

	B	
	290	65
20	275	
60	195	
	190	24
68	120	
55	45	80
	A	
GO	S.W.	
Q5		

	B	
	56	
21.3	40.6	
	32	18.8
	18	22
8.5	14.2	
	A	
Q7		



Q18

10 per cent in bulk when loosely piled. The earth is stacked on level ground at the side of the trench in a heap, the cross-section of which is an isosceles triangle of base 6 ft. Make a complete drawing of the section of the trench and piled earth to a scale of $\frac{3}{4}$ in. to 1 ft. (Ref. Fig. 114.)

17. A piece of lead triangular in shape, and weighing 5 lb. has to be cut into strips each of which must weigh 1 lb. Four of the strips must have two sides parallel and the fifth must be a triangle. Set out the original triangle to scale making the sides 3 in., $2\frac{1}{2}$ in. and 2 in. long respectively, and divide it in accordance with instructions. (Ref. Fig. 121.)

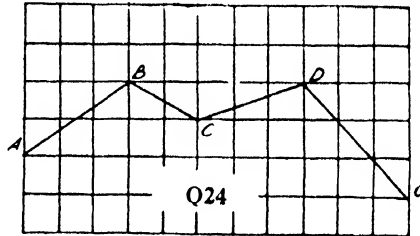
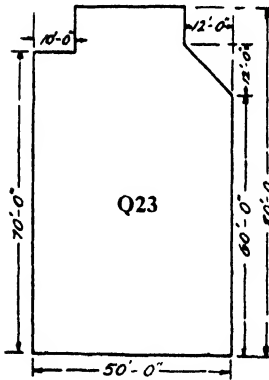
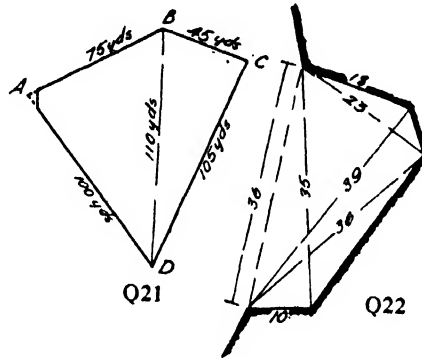
18. A half-timbered gable is shown in the figure. The panels between the uprights (A B C D) are approximately equal in area. Set out the gable to a scale of $\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 123.)

19. Draw a triangle ABC given sides $AB = 3\frac{1}{2}$ in., $BC = 3\frac{1}{4}$ in., $CA = 2\frac{1}{2}$ in. Mark a point D on the side AB, $\frac{3}{4}$ in. from A. This figure represents the plan of a piece of land with a house D, drawn to a scale of 1 in. to 100 yds. The land has to be divided into equal areas by a straight fence from D. Set out the line of fence. (Ref. Fig. 124.)

20. If the land in the previous exercise had to be divided from D into four equal areas, set out the fence lines.

21. Set out the building site ABCD to a scale of $\frac{1}{2}$ in. to 10 yds. The site has to be divided into three equal parts by means of brick walls terminating at the entrance gate A. Draw the complete plan of the plots. (*Hint*: Transform figure to a triangle with vertex A.)

22. Set out the irregular floor space to the dimensions shown, and find the number of



squares of floor boarding required to cover the space, allowing 5 per cent for waste. The measurements are in yards.

23. The plan of a small hall is given. Draw this to a scale of $\frac{1}{2}$ in. to 10 ft., and find its area graphically. 20 per cent of the floor space is occupied by the aisles, etc., and of the remaining area each person is allowed 4 sq. ft. How many persons will the hall accommodate?

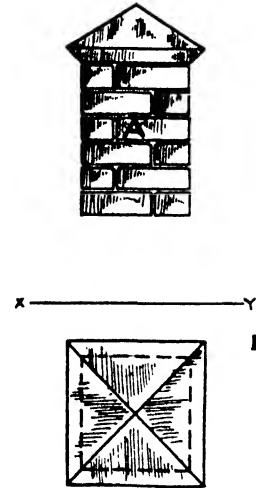
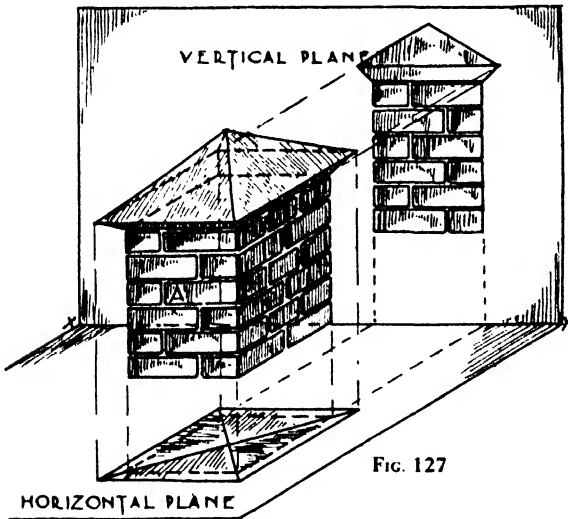
24. A rectangular field with a zigzag fence is shown in the figure. The squares (which are for setting out purposes) represent 10 yds. It is required to erect a straight fence from O to displace the existing fence without altering the areas of the two plots. Re-divide the field. (*Hint*: Commencing at A, substitute one line for AB and BC without alteration of areas, and work towards O.)

CHAPTER VII

Plans, Elevations, Sections

Plain Orthogonal Projection

You are perhaps familiar with the ordinary drawings of simple objects, i.e. plans, elevations, and sections. This type of drawing is the one usually adopted in the manual instruction centre, as a preliminary to the making of any particular object. Evening continuation schools also include this type of projection in their syllabuses. In building, engineering, and other branches of industry these projections are known as *working drawings*. The drawings are made in the architect's or engineer's drawing office, and are sent into the workshops and on to the buildings for the artisans' use. The correct name for these projections is *orthogonal* (sometimes called *orthographic*).



If you have attended either a manual instruction centre or an evening continuation school, the elementary principles of projection will no doubt be familiar to you, so we will just refer to Fig. 127 for a moment and recapitulate. The object depicted is the upper portion of a gate pier. The vertical plane (V.P.) contains the elevation of the object, and the horizontal plane (H.P.) the plan. Notice particularly that the elevation consists of a front view of face A, i.e. *the face nearest the point of vision, and farthest from the plane*. Similarly, the plan depicts the top face of the object, again the face farthest from the plane.

The "XY line" is not only the dividing line between the horizontal and vertical planes, but it is also the elevation of the horizontal plane and the plan of the vertical plane. This fact must be borne in mind when dealing with the problems on auxiliary elevations which follow. Fig. 128 represents the plan and elevation of the pier as they would actually appear on our drawing paper. In orthogonal projections, we have to *imagine* our paper folded along the XY line in order to obtain the two planes. In drawings of elaborate objects such

as buildings, many elevations, plans, and sections are necessary, and as it would be impossible actually to fold our paper in order to obtain a particular plan or elevation, imagination must of necessity play an important part.

We will now proceed a step further.

Fig. 129 is a pictorial view of a quoin brick in space, with the V.P., H.P., and S.P. (side plane).

The view on the side plane is taken from the direction of arrow A. If we lay the planes out in one plane along the lines XY and XZ, the views appear as in Fig. 129A. The width of the object on the S.P. you will notice is the same as the width on the H.P. It would be permissible to lay out the side plane as shown in Fig. 129B, as this follows the principles of projection, the only difference being that whereas in Fig. 129A the side elevation is projected direct from the front elevation, in Fig. 129B it is projected direct from the plan.

Auxiliary Projections

In many building drawings, elevations other than front and side elevations are necessary. For instance, Fig. 130 represents the plan of building at the corner of a road. You will observe that neither an elevation taken from arrow L, nor one taken from arrow M will truly show the details in the wall AB. This wall may contain door and window openings, rain-water heads, etc., in fact, many details, the true form of which it is necessary to reproduce on the drawings. To show these details we must make an elevation parallel to the face AB or perpendicular to the direction of the arrow N. Such a view would be termed an *auxiliary elevation*.

Auxiliary Elevations

Fig. 131 is a pictorial view of a simple auxiliary projection of a triangle in space. The plane of the triangle CDE is vertical. Its plan is a straight line *cde*, and its elevation on the V.P. is C'D'E'. As the plan of the triangle is inclined to the V.P., C'D'E' will not be the true shape of the figure. To obtain a true view of the triangle, we must draw an elevation on another V.P. which is parallel to the plane of the triangle, i.e. parallel to the plan. This plane (A.P.) is not perpendicular to the original V.P., the angle at Z being greater than 90°, therefore the dividing line (X'Y) between H.P. and A.P. is not at right angles to XY. To draw the auxiliary elevation of the triangle on A.P. the usual rules of projection must be followed. All projections from the plan to the elevation must be perpendicular to the new X'Y. The heights of the corners of the triangle above H.P. remain the same, of course, on the new elevation, i.e. C"D"E" are the same heights respectively above X'Y as C'D'E' are above XY.

We will now draw the same triangle in orthogonal projection (Fig. 132). First draw the plan and elevation of the figure below and above XY respectively, in the usual way. The auxiliary plane (represented by X'Y') is then drawn parallel to *cde* at any distance from the plan. Project *cde* perpendicularly to the new plane, and make the heights of C"D"E" above X'Y' respectively similar to the heights of C'D'E' above XY. E.g. E"M" = E'M'. We can summarize the rules for drawing auxiliary elevations in the following words: *Draw projectors from all points in plan perpendicular to the new ground line, and make all heights in the new elevation correspond to the heights of the same points in the original elevation.*

Auxiliary elevations may be projected from any direction. Fig. 133 represents a "straight" plan and elevation, and two auxiliary elevations of the roof of a square tower.

The construction should not be difficult to follow if the rule stated in the last paragraph is adhered to. Note that the heights of all points in the auxiliary views are the same as the heights of the corresponding points in the original elevation A. E.g. V' and V'' are the same heights above their respective ground lines as V is above XY .

Auxiliary views of circular objects are treated in the same manner as the auxiliary projections of straight-sided objects.

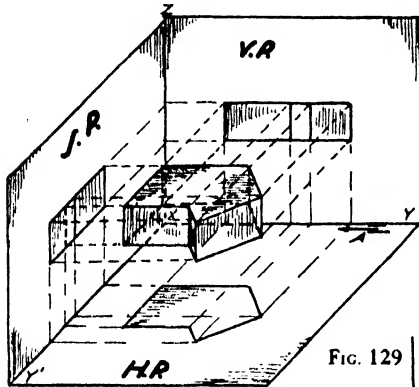


FIG. 129

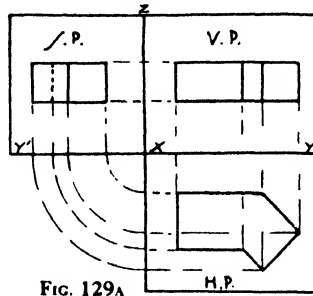


FIG. 129A

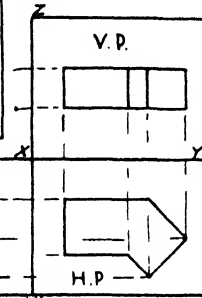


FIG. 129B

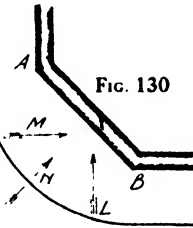


FIG. 130

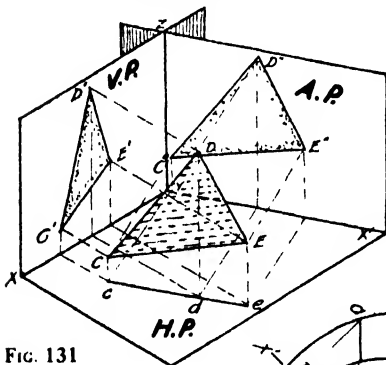


FIG. 131

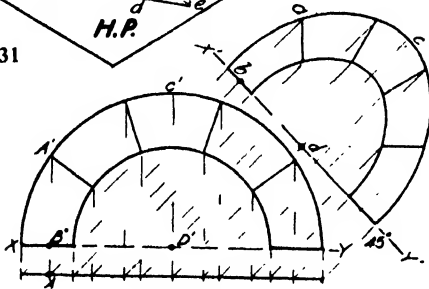


FIG. 134

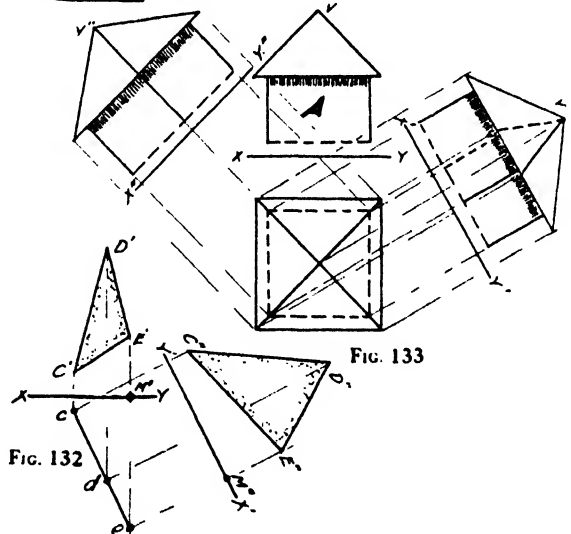


FIG. 133

EXAMPLE. (Fig. 134.) *Given the plan and elevation of the face of a semicircular stone arch. A projection of the arch as viewed from an angle of 45° with the wall face is required.*

Project all points in the elevation into plan. Draw a new $X'Y'$ at an angle of 45° with the plan and project above this $X'Y'$ all points from the plan. Make the heights of the points above $X'Y'$ similar to the heights of the corresponding points above XY . E.g. $ab = A'B'$, and $cd = C'D'$.

Mitres

EXAMPLE. (Fig. 135.) *The section and plan of a length of mitred moulding is given. It is required to find the true shape of the mitre.*

Project all points in the elevation into the plan (including several points placed on the curved portion of the mould).

Draw an $X'Y'$ parallel to the plan of the mitre CD , and project perpendicularly all points from plan into the auxiliary plane above $X'Y'$. Make the heights of the various points in the new elevation correspond to the heights of the same points in the original elevation, i.e. make $h = H$ and so on.

EXAMPLE. (Fig. 136.) *The outline of the plan of a corner of a room and cornice mould is shown. EF and CD are the plans of rough wood brackets used for supporting the plaster cornice. The plan and true shape of the mitre bracket AB is also shown. The true shape of the ordinary brackets CD and EF is required.*

Project the points on the outline of the mitre bracket perpendicularly on to AB . Project all points thus obtained on AB perpendicularly to CD . Make $DD' = AA'$, $gg' = GG'$, $gg'' = GG''$ and so on.

EXERCISES

1. The plan of a roof is given, the roof slope being 30° . Draw front and end elevations to a scale of 1 in. to 10 ft. (*Hint*: Draw an XY line parallel to CD to obtain height of ridge.)

2. Draw a new elevation of the roof in Exercise 1 so as to show the true shape of the gable AB . (*Hint*: Draw a new ground line parallel to AB .)

3. The plan of a square chimney shaft protruding through the ridge of a roof pitched at 45° is given. The shaft is 4 ft. high above the ridge. Draw a front and side elevation of the roof and shaft. Scale $\frac{1}{2}$ in. to 1 ft. (*Hint*: Draw side elevation first on XY taken perpendicularly to ridge.)

4. Assume the chimney shaft in Exercise 3 to have two of its faces *parallel* to the ridge, and follow the wording of Exercise 3.

5. The diagram is the elevation of a hexagonal turret roof. Draw the plan, and an elevation as viewed from a direction of 45° with the existing viewpoint. Scale $\frac{1}{8}$ in. to 1 ft. (Ref. Fig. 133.)

6. The base of an octagonal pier is shown in elevation. Draw the plan, and an auxiliary elevation taken from a viewpoint which is at 30° with the face of the pier. Scale 1 in. to 1 ft.

7. The plan of a groined roof is given. The section of AB is a semicircle. Draw:
(a) The section at CD .
(b) The shape of the rib EF .

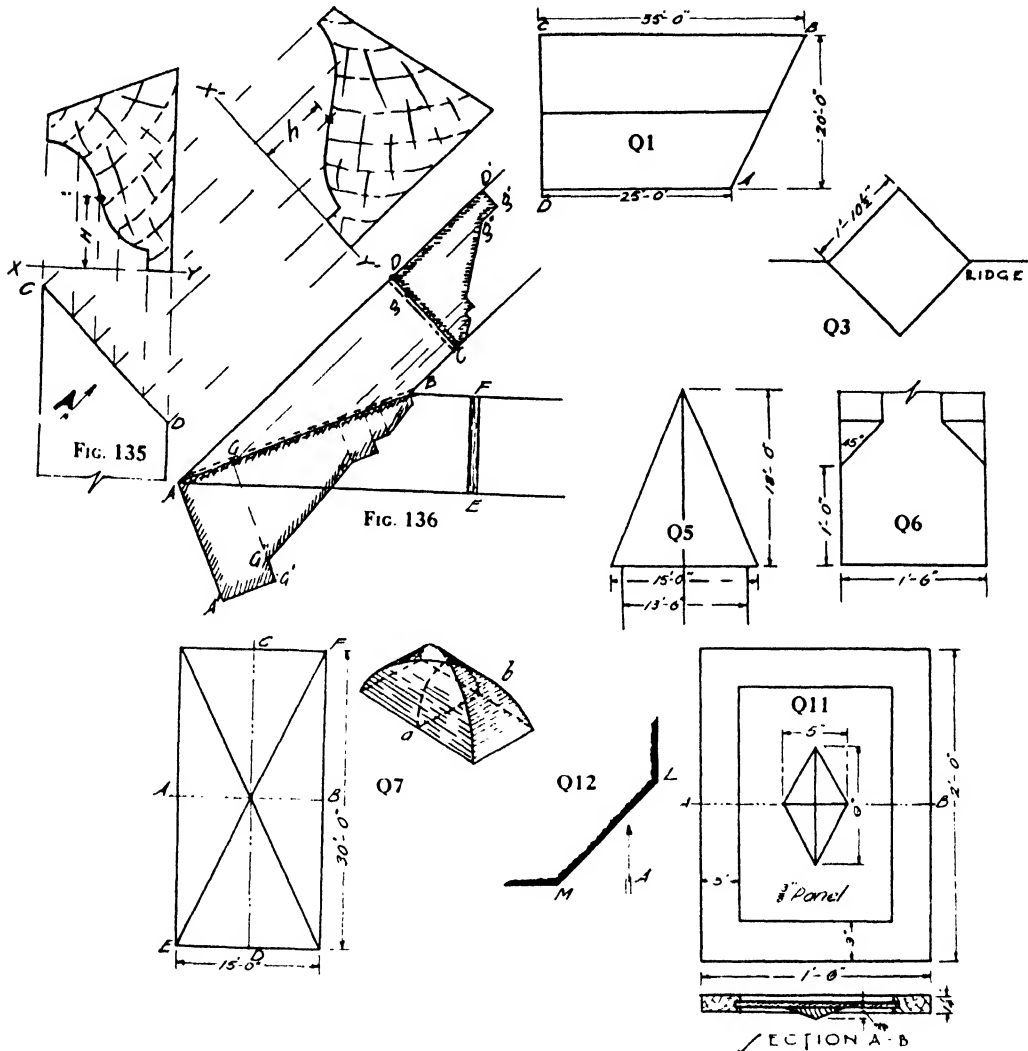
(Scale 1 in. to 10 ft.). (*Hint*: Both section CD and rib EF will be semi-ellipses with half minor axes equal to radius of semicircle on AB .)

8. A rectangular building 50 ft. by 25 ft. is covered with a hipped roof. The larger roof surfaces are inclined at 30° to the horizontal, and the hipped ends are inclined at 22° . Draw the plan of the roof to a scale of $\frac{3}{4}$ in. to 10 ft., and an elevation viewed from such a position that two of the hip rafters appear to be vertical. What is the true length of a hip rafter? (*Hint*: Obtain height of ridge by drawing a cross-section first. Follow with front elevation, and finally the plan.)

9. If the building in the previous question had the two hipped surfaces of the roof inclined at 45° to the horizontal, one of the main roof surfaces inclined at 30° and the other main

surface at 45° , how would the plan then appear? Draw this plan, and two elevations that show the true lengths of the hip rafters. What is the height of the ridge above the eaves level? (*Hint*: Proceed as in Question 8.)

10. A four-panelled door has the following dimensions in elevation: Height 6 ft. 6 in., width 2 ft. 6 in., bottom and lock rails 9 in. wide, top rail, stiles and muntins $4\frac{1}{2}$ in. wide, centre of lock rail 2 ft. 9 in. from bottom edge of door. Draw the elevation of the door



(a) when closed, (b) when open at an angle of 45° . (Thickness of the door is 2 in.) Scale 1 in. to 1 ft.

11. The elevation and section of a small door are given. Draw these views, and a further elevation of the door when it is swung open at an angle of 35° with its closed position. Scale $1\frac{1}{2}$ in. to 1 ft. (*Hint*: Draw new XY at 35° with sectional plan.)

12. The figure represents the plan of the face of a corner of a building. The wall LM contains an opening which appears to be a circle of 4 ft. dia. when viewed from the direction A. Draw the true shape of the opening to a scale of $\frac{1}{2}$ in. to 1 ft.

13. The squares in the diagram have $\frac{1}{2}$ in. sides, and are inserted to enable you to set out the figure. The section of an architrave moulding and the plan of the block on which it rests are given. The upper end of the block is splayed from back to front (i.e. from *a* downwards to *b*) at an angle of 45° . Draw the side and front elevations of the moulding and block in contact. Scale half full size. (*Hint*: First draw side elevation, i.e., a view looking in direction of arrow.)

14. A cove mould of the design shown in Fig. 100A intersects a similar mould at an angle of 75° . Draw the true shape of the mitre.

15. A horizontal circular shaft 3 ft. dia. has to pierce a wall at an angle of 60° with the face of the wall. Draw the true shape of the templet required for cutting the hole in the wall.

Auxiliary Plans

As new elevations can be drawn on vertical planes erected (to suit our convenience) at any angle to the original vertical plane, so new plans may be drawn on planes that make any angle to the horizontal plane.

Auxiliary plans are useful for finding the true shapes of surfaces inclined to the horizontal and for many other purposes.

Fig. 137 is a pictorial view of a king closer in space, with a plan, elevation, and auxiliary plan. Fig. 137A is the orthogonal projection of the same object.

When drawing auxiliary elevations, projections from the original plan are taken, and the heights of the corners of the object above the auxiliary XY are measured from the original elevation. In auxiliary plans this procedure is reversed; the rule is, *project from elevation and make the distances below the new X'Y' the same as the distances of the corresponding points below the H.P.* E.g. in Fig. 137A $AB = a'b'$ and $CD = c'd'$. Reference to Fig. 137 will illustrate this more clearly. By introducing a new plane inclined to the H.P., we do not alter the distance of the object from the V.P. In point of fact, the same V.P. is utilized for both plans, hence the same relative position of the object with the V.P. Turn your book around until the X'Y' in Fig. 137A is horizontal. Now you see the closer apparently inclined to the H.P. as represented by X'Y'. In reality, the object has been left in its original position, and a new plane (X'Y') has been introduced to suit the new point of vision, i.e. from a direction V.

EXAMPLE. (Fig. 138.) *The plan and elevation of a hipped roof are given. A plan that will show the true shape of surface A is required.*

To see the true shape of the hipped end A, we must view the roof perpendicularly to A, i.e. from the direction of R. Draw X'Y' parallel to B'C', and project (perpendicularly) all corners of the elevation below X'Y'. Locate points on these projectors below X'Y' to correspond with the same points below XY. E.g. $c'c'' = cC$, $b'd = B'B$, $d'd = DB'$, and so on. A' will be the true shape of the surface A.

EXAMPLE. (Fig. 139.) *To draw a new plan of the quoin brick when the edges AB rest on the ground.*

Draw an X'Y' parallel to the line passing through A'B'. Now proceed as in the last Example, i.e. make $W' = W$ and $w' = w$. Complete the plan as shown.

EXAMPLE. (Fig. 140.) *An octagonal shaft protrudes through a roof as shown. To draw the shape of the templet required for cutting the hole in the roof.*

Make the new $X'Y'$ parallel to the roof slope and imagine the hole to be viewed from the direction of arrow V . Proceed as in previous Examples, after drawing a temporary XY . I.e. make $aa' = AA'$ and so on.

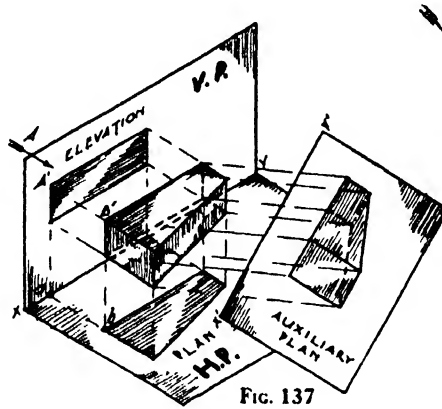
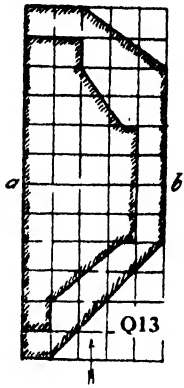


FIG. 137

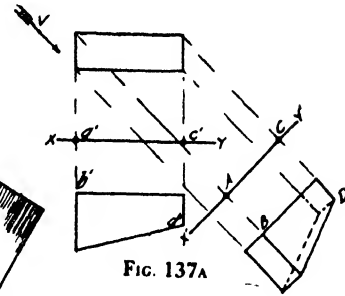


FIG. 137A

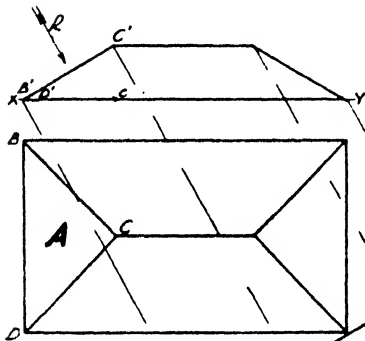


FIG. 138

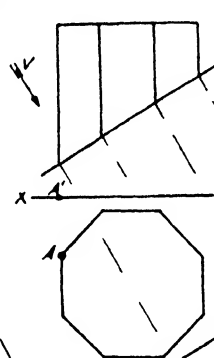


FIG. 140

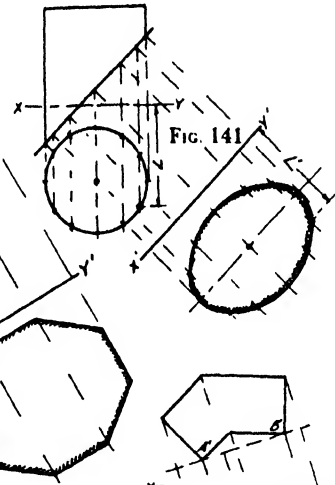


FIG. 141

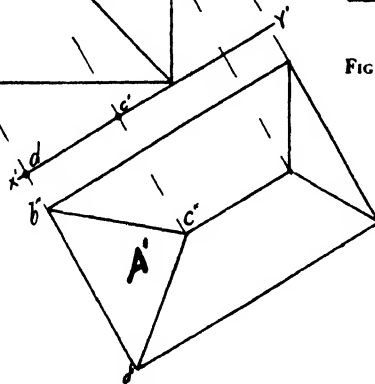


FIG. 139

EXAMPLE. (Fig. 141.) Similar to the preceding example with the word "circular" substituted for the word "octagonal."

Divide the circular plan into any convenient number of equal parts ; project these points on to the roof slope, and proceed as in the previous Examples.

Sectional Plans and Elevations

The drawing of sections, sectional plans, and sectional elevations involves the same principles as those already illustrated in this chapter. We had better be quite clear as to the difference between a section and a sectional plan or sectional elevation.

Fig. 142 represents a stone cap in orthogonal projection, with a vertical cutting plane HT. View A is a sectional elevation and views C and D are sections. A section is a view of the cut face only; a sectional elevation not only shows the cut face, but every part of the solid lying behind that face.

To draw the section (or sectional elevation) imagine the part of the object in front of the cutting plane (i.e. nearest the eye) removed, and an elevation taken of the remaining part.

In Fig. 142 the cutting plane HT intersects the base lines at E and G, therefore project E and G to E' and G' . The plane intersects hj at F, so project F to $h'j'$ (the same line in elevation).

Repeat this construction for all points of intersection between plane and solid. View A is thus obtained. The section in view C is identical with that in A. View D is the *true* shape of the section and is obtained in a similar manner to that for the auxiliary elevations, all heights above $X''Y''$ being taken from view A.

EXAMPLE. (Fig. 143.) *Plan and end elevation of a ridge tile is given. To draw a sectional elevation on HT.*

The section made by cutting plane HT on XY, will be similar to the elevation on XY. Draw a new $X'Y'$ parallel to the cutting plane HT and project all points of intersection between cutting plane and plan into the new plane $X'Y'$. Make the heights of all points above $X'Y'$ similar to the heights of the corresponding points above XY, e.g. $a'a = A'A''$, and so on.

Complete the sectional elevation as shown.

EXAMPLE. (Fig. 144.) *Given two views of a lamp shade, and cutting plane VT. To draw (a) a sectional plan, (b) a sectional plan to show the true shape of the section.*

(a) Project the points of intersection of cutting plane VT and the object on to the corresponding lines in the plan at ABCD.

(b) Draw a new $X'Y'$ parallel to VT and project all points on and below VT into the new plane below $X'Y'$. Locate the angles of the object below $X'Y'$ by making their distances below $X'Y'$ correspond to their respective distances below XY, e.g. make $ce = CE$ and so on for all points.

EXERCISES (Contd.)

16. The plan of a plot of ground which slopes upwards at 30° (from left to right) is given. Draw the true shape of the plot, and find the area graphically. Scale $\frac{1}{8}$ in. to 10 yds.

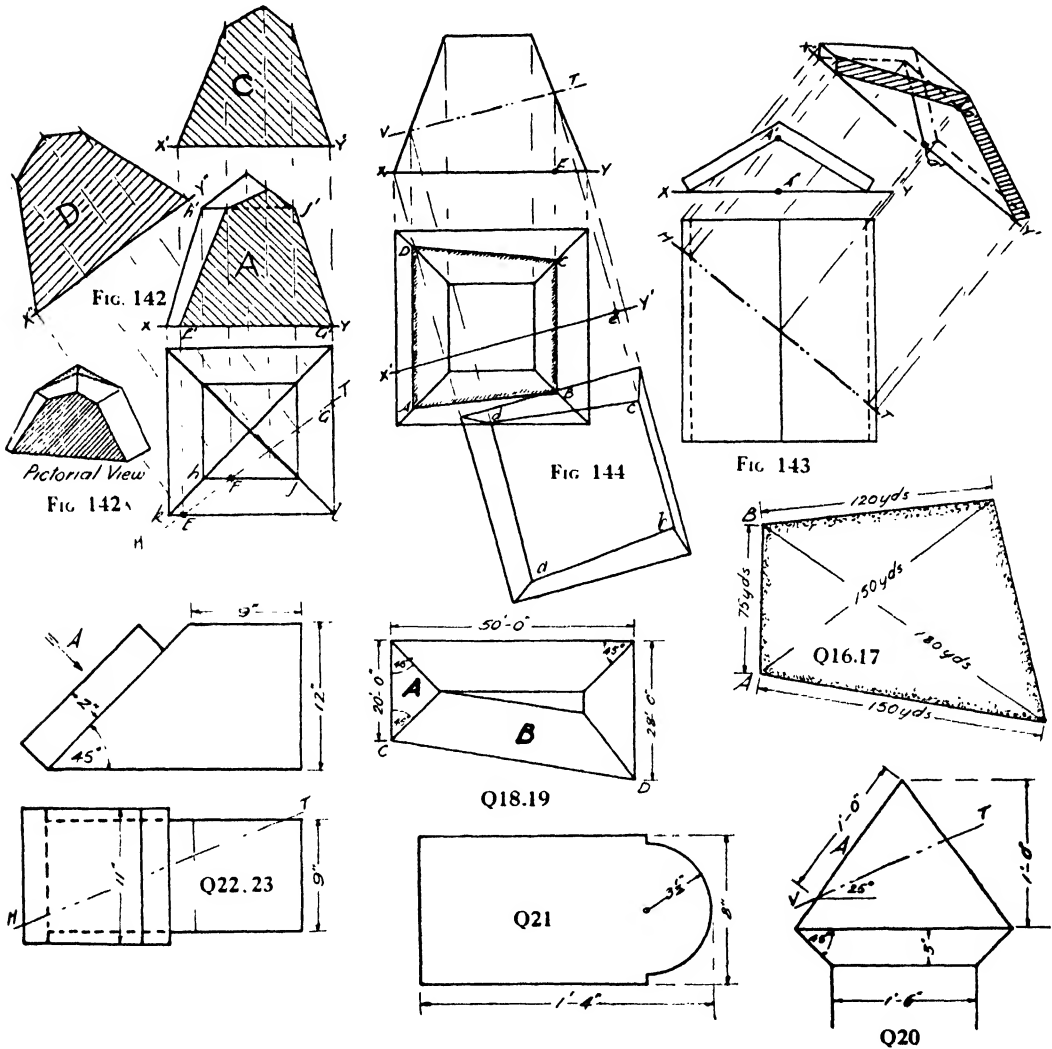
17. If the diagram in Exercise 16 was the true shape of the plot sloping upwards at 30° from left to right, what would be the actual shape in plan?

18. All the roof surfaces in the figure are inclined at 30° to the horizontal. Draw a plan of the roof to show the true shape of face A. Scale $\frac{1}{16}$ in. to 1 ft. (*Hint*: First draw an elevation on an XY taken parallel to 50 ft. eave.)

19. Draw a new plan of the roof in Exercise 18 to show the true area of surface B. (*Hint*: Draw an elevation on XY taken perpendicularly to CD.)

20. (a) Draw an auxiliary plan of the square stone cap to show the true shape of the face A. (b) Draw a sectional plan of the cap taken perpendicularly to the cutting plane VT. Scale 1 in. to 1 ft. (Ref. Figs. 138 and 144.)

(Note: Hints and references have been omitted purposely in the following questions.)



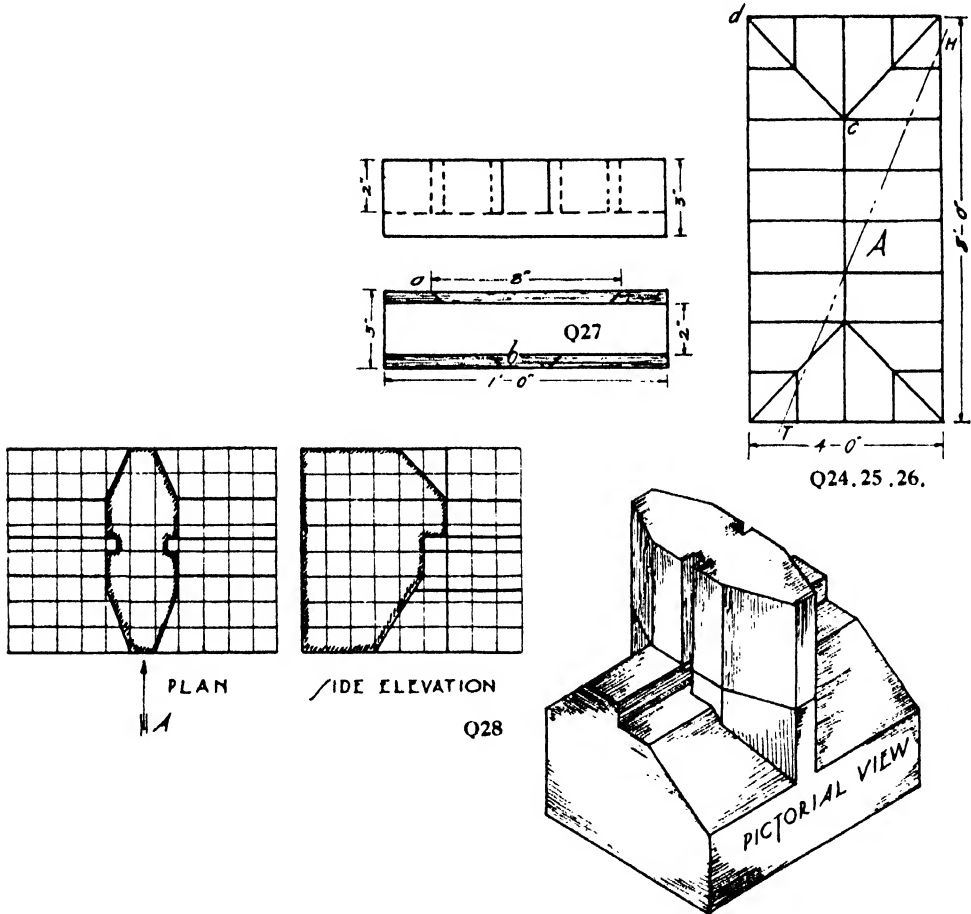
21. Draw the plan of the ornamental roof tile when laid on a roof pitched at 45° . Scale 2 in. to 1 ft.

22. Draw a new plan of the kneeler when viewed from the direction of the arrow A. Scale one-eighth full size.

23. Draw a sectional elevation of the kneeler in Exercise 22 on a vertical plane taken parallel to the vertical cutting plane HT.

24. The outline of the roof of a lantern light is given in the diagram. Draw an auxiliary plan to show the real shape of face A. The roof surfaces slope at an angle of 30° to the horizontal. Scale $\frac{3}{8}$ in. to 1 ft.

25. Draw an elevation of the lantern roof in Question 24 in such a position that the hip bar cd appears as a vertical line.



26. Draw a sectional elevation of the lantern roof in Question 24 on a V.P. erected parallel to the cutting plane HT.

27. A block used in the cutting of 45° mitres is shown in the sketch. Draw a sectional elevation of the block on an XY taken parallel to the mitre cut ab . Scale quarter full size.

28. The squares in the figure have sides of $\frac{1}{4}$ in. and are inserted to aid your setting out.

Two views of the joint between the sill and mullion of a stone window are given. Draw a view looking from the direction of the arrow A, showing hidden lines dotted. Scale quarter full size.

CHAPTER VIII

Pictorial Projection

General Principles

Under the heading of "Pictorial Projection," there are many types of views including several which are of the greatest importance to the building student. At this stage, however, it is not desirable to consider all the various pictorial views it is possible to depict on drawing paper; it will be sufficient to mention some by name only, and study the others. The main difference between a pictorial view of an object and an orthogonal view (or working drawing) is, whereas the former view shows more than one face of the object *on a single drawing*, the orthogonal view depicts one face of the object only on the one drawing, so a complete orthogonal projection of any object must of necessity contain more than one view. To understand this fully, refer to Fig. 145A and B. Fig. 145A is a pictorial view of a king closer and Fig. 145B an orthogonal view of the same object.

In the pictorial projection three surfaces of the closer are obtained on the one drawing, the top A, the front face B and one end C. In the working drawing, Fig. 145B, three separate

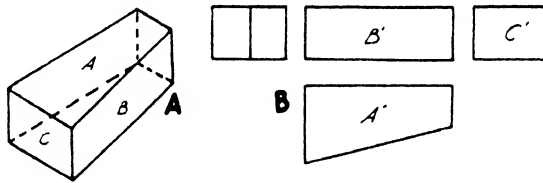


FIG 145

drawings are required to obtain views of the same three surfaces. Both types of projection have their own particular uses, and it would not be practicable to substitute one for the other in actual building work. Pictorial views do not show the *true shape* of the object, therefore it would not be advisable to make the drawings from which the builder has to work in pictorial projection. On the other hand, the average person without technical knowledge would find it difficult to visualize the appearance of a building in orthogonal projection, simply by reference to views marked "Plan," "South Elevation," "North Elevation," etc. Such views convey little to the mind of the layman who may wish to visualize the complete structure before it is built.

Various Types

The chief types of pictorial views are as follows : *Isometric, Oblique, Axonometric, Planometric and Perspective.*

Figs. 146 to 150 depict a portion of a chimney breast in each of these types of projection.

Isometric, oblique, planometric and axonometric are *parallel* projections, i.e. they are formed by parallel lines. Perspective projection consists of lines which if continued would converge to various points. This type of projection is more natural than the others mentioned, but the construction is somewhat difficult and is beyond the scope of this book.

If you compare the four parallel projections, you will notice perhaps points of similarity between isometric and axonometric, and between oblique and planometric. Again, planometric and axonometric are similar inasmuch as the top surface of the chimney breast (which

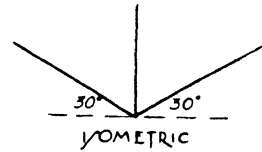
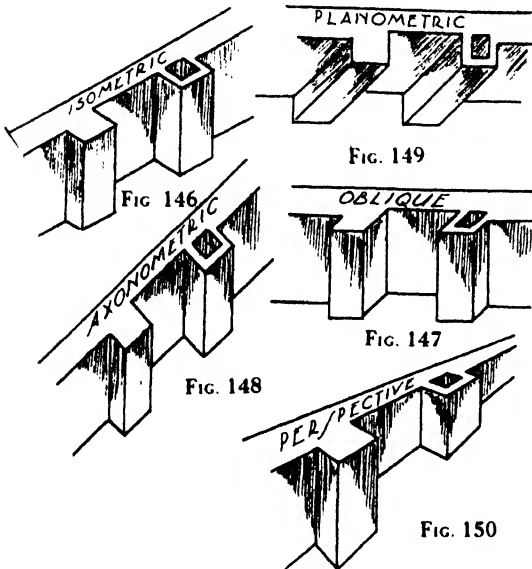


FIG 151

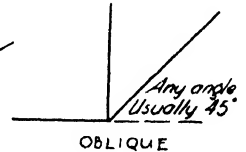


FIG 152

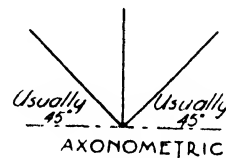


FIG 153

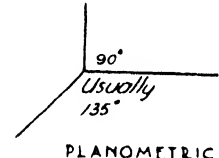
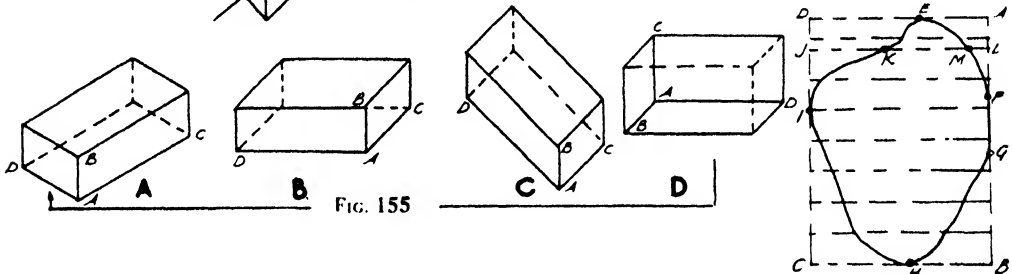


FIG 154



we know actually consists of several rectangles) is *truly* depicted in both views. Whereas in both isometric and oblique, this surface is distorted, consisting of a series of parallelograms.

Isometric and oblique projections are useful for rapid sketching where a comprehensive view of the complete object is required. The other two parallel projections would produce

a less restricted view of the *interior* of a building if a drawing were required on which internal details were to be shown. Do not think, though, that axonometric and planometric are used exclusively for interior views. Axonometric in particular lends itself to external views with little distortion.

The axes of the four parallel projections are shown in Figs. 151 to 154. Always commence the drawings with these axes.

Regular and Irregular Figures and Solids

EXAMPLE. (Fig. 155.) *To draw a $9'' \times 4\frac{1}{2}'' \times 3''$ brick in (A) isometric, (B) oblique (C) axonometric, (D) planometric.*

In each projection commence by drawing the appropriate axes AB, AC, and AD. Measure the thickness of the brick (3 in.) along AB, the width ($4\frac{1}{2}$ in.) along AC and the length (9 in.) along AD. Now complete the figures by drawing the remaining lines parallel to the axes.

As the principles underlying the construction of the four parallel projections are similar, we shall not deal with all the projections in every example, but we shall try to grasp these principles in any one of the projections, in order that we may be able to apply those principles to the others.

EXAMPLE. (Fig. 156.) *To draw any straight sided figure in isometric projection.*

Enclose the figure in a rectangle the sides of which touch the extremities of the figure. Now draw the rectangle in isometric projection. To locate points G, F, C, B, and A in the isometric, make $ng = NG$, $nf = NF$, $lb = LB$ and so on.

E and D are located in the isometric view by reproducing the rectangles HKJE and IKOD in isometric in their correct positions relative to the main rectangle, i.e. make $ik = IK$, $ko = KO$ and so on.

To complete the figure connect the points thus obtained.

We will now apply these principles to a curved outline.

EXAMPLE. (Fig. 157.) *To draw an irregular curved plot of land in pictorial projection, say oblique.*

Enclose the plot in a rectangle, and divide this rectangle into any number of equal strips. Generally speaking, the more irregular the shape, the greater will be the number of ordinates required. Reproduce the rectangle and ordinates in oblique projection. Locate the points of contact between rectangle and figure EFGHI in the pictorial view in the manner described in the last Example. Now obtain other points in the pictorial view by measuring the distances from the end of each ordinate to the point of intersection of ordinate and curve, and transferring these points to the pictorial view. E.g. make $jk = JK$, $lm = LM$, and so on. Trace a curve through the points thus obtained. If any irregularity occurs in the curve, an extra ordinate may always be inserted. E.g. in the figure, perhaps it would be advisable to insert an ordinate between A and L, as the curve does not follow its customary course between K and E.

Solids with three dimensions will now be considered more fully.

EXAMPLE. (Fig. 158.) *To draw the box in axonometric projection.*

Draw the outside edges of the box to the proper dimensions in axonometric projection as illustrated in the drawing of the brick (Fig. 155). The inside of the box in the pictorial view is reproduced in a similar manner, i.e. $gh = G'H'$, $gj = G'J'$ and so on. Fig. 159 is an isometric view of the same box. Notice that a more restricted view of the inside of the box is obtained in this type of projection.

Various Building Objects

EXAMPLE. (Fig. 160.) *To draw the putty tub in isometric projection.*

Commence by drawing the base IJLK in isometric. In the centre of this base, erect a square prism of side AD and height A'A". This would be an isometric view of the block from which the tub could be cut assuming it were solid. In the centre of *ijkl* construct the isometric view of the square of GE side; to do this, draw a square *m'emi* of side EM in isometric to locate one corner. Now join the outside edges *de*, *ag*, etc. Draw the small square at C in isometric, and thus obtain the thickness of the tub sides. A small

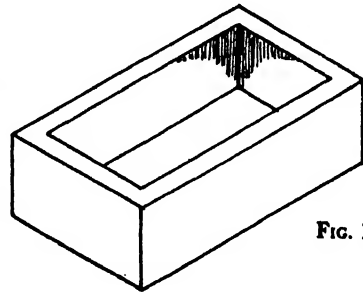
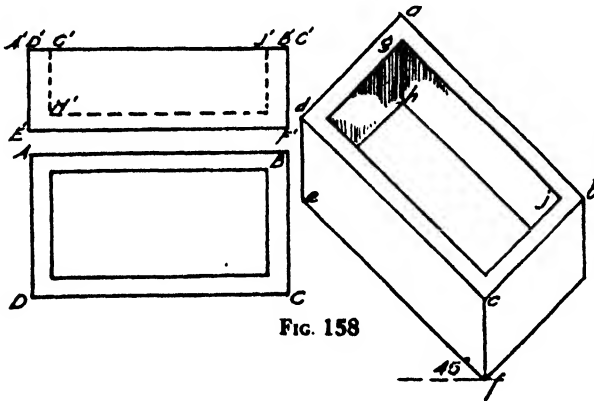


Fig. 159

isometric square in the middle of *efgh* equal in size to the smallest square in the plan will enable you to complete the isometric view.

A circle appears to be an ellipse when drawn in isometric projection.

EXAMPLE. (Figs. 161A and 161B.) *Plan and elevation of a semicircular stone arch are given. (a) Isometric and (b) oblique views of the arch are required.*

(a) Enclose the elevation of the arch in a rectangle A'EFG, and draw this rectangle in isometric *aefg*. Draw parallel ordinates from all points in the arch in elevation to the sides of the rectangle A'EFG, and reproduce these ordinates in the isometric view. All points on the arch face are thus obtained in isometric. Trace a fair curve through the points, and draw the joint lines of the voussoirs. Make $ak = A'K'$ and the other joints *lm*, etc., the same length. The back of the arch can then be drawn.

(b) The oblique view is quite simple. Reproduce the arch face as in the front elevation, and then draw the back part of the arch by making $a'k'$, $l'm'$, etc., each equal to $A'K'$, and sloping them, of course, at say 45° .

Figs. 163 to 166 show four pictorial views of the stone gate pier cap (Fig. 162). The construction should not be difficult to follow. Draw the square prism ABCD in the type of projection required, and divide it at EF and GH. Make $kl = KL$ in each case. The projections then follow the rules stated in previous examples. V in each case is the centre point of the uppermost square.

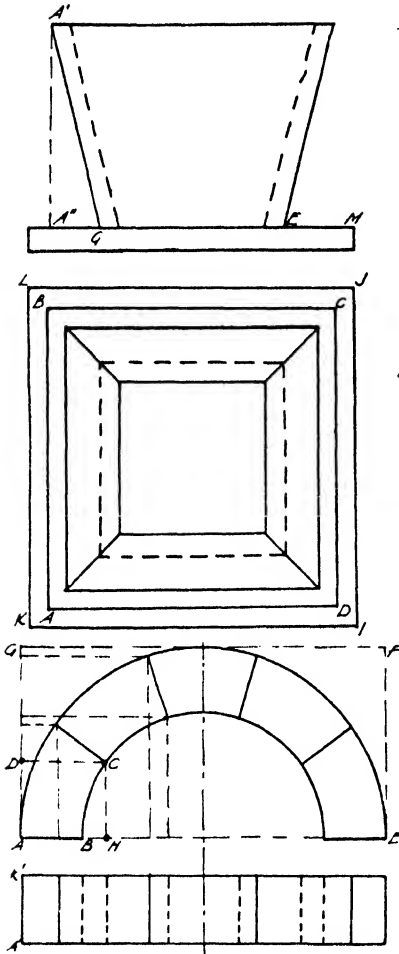


FIG. 161

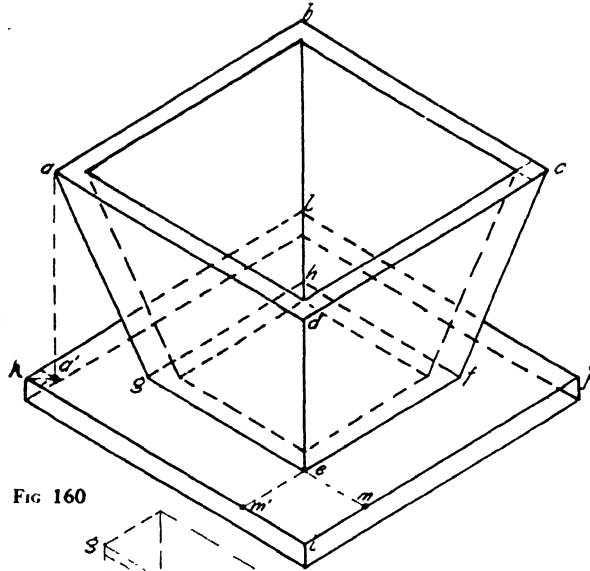


FIG. 160

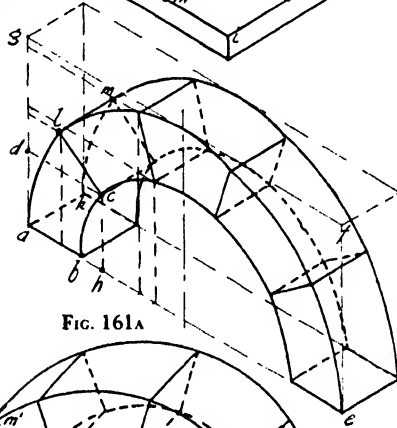


FIG. 161A

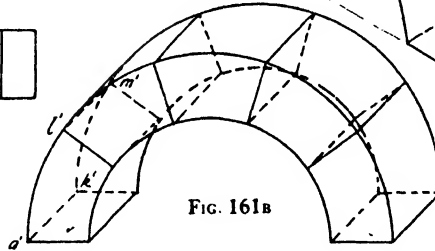


FIG. 161B

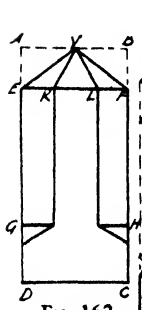


FIG. 162

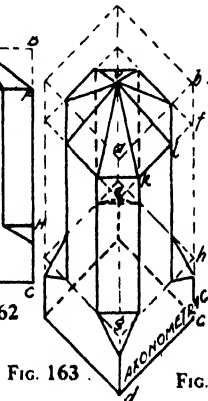


FIG. 163

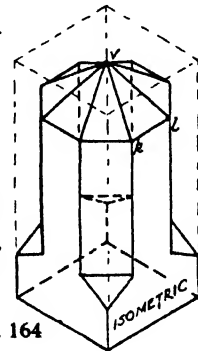


FIG. 164

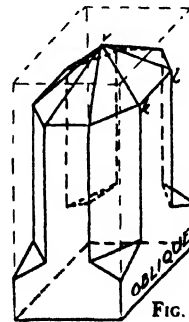


FIG. 165

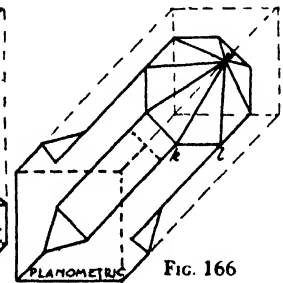


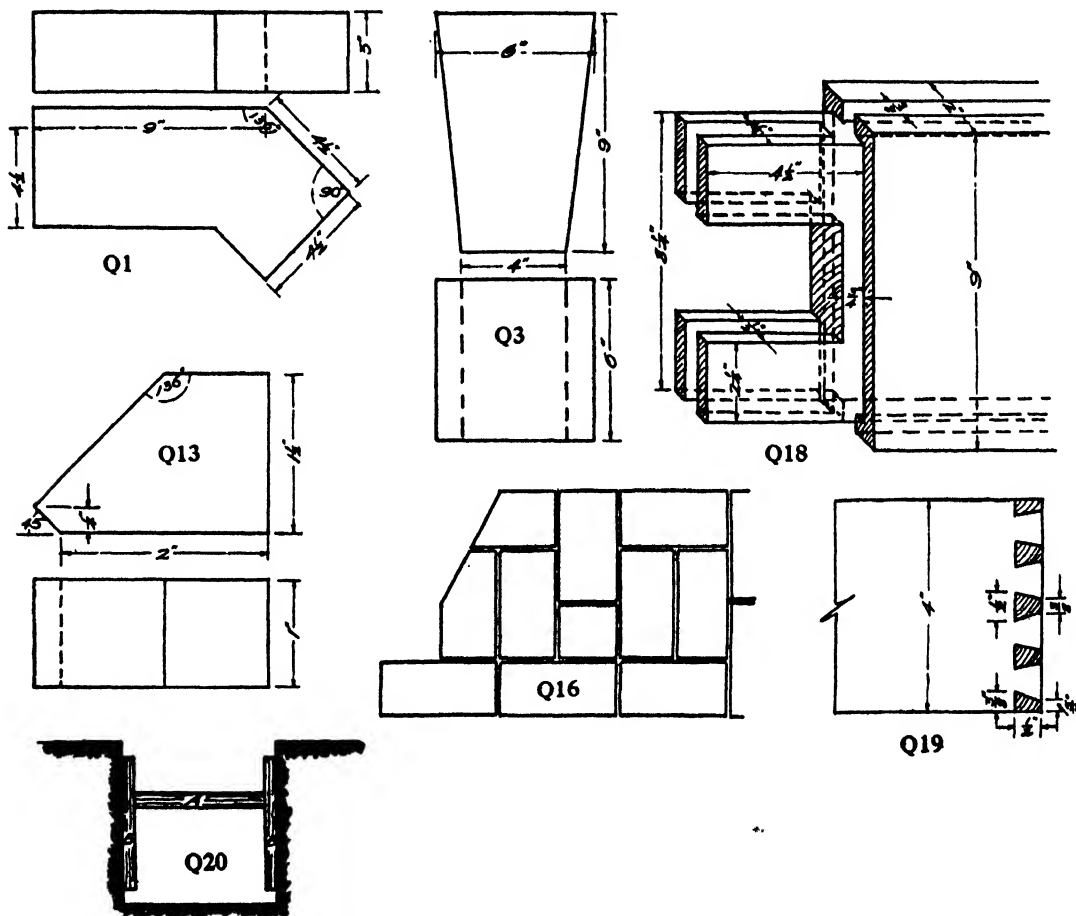
FIG. 166

EXERCISES

1. Draw the quoin brick in four different types of pictorial projection. Scale quarter full size. (*Hint*: Enclose the plan in a rectangle.)

2. Draw the mitre block in Question 27, p. 66, in isometric and oblique projection. Scale quarter full size.

3. Make an axonometric projection of the keystone. Scale quarter full size.



4. Draw the lamp shade in Fig. 144 in planometric projection. Assume the two squares in plan to have sides of $1\frac{1}{2}$ in. and $2\frac{1}{2}$ in. respectively, and the height of the elevation to be 2 in.

5. Assume the stone cap in Fig. 142 to have the following dimensions: $kl = 2$ in., $hj = 1$ in., height to $j' = 1\frac{3}{4}$ in., total height = $2\frac{1}{2}$ in. Draw the cap in isometric projection.

6. Draw the hipped roof, Fig. 138, in axonometric projection. Make the length 3 in., the width $1\frac{1}{2}$ in. and the vertical height from eaves level to ridge $\frac{7}{8}$ in.

7. Draw the ridge tile, Fig. 143, in oblique projection. The dimensions are as follows: Tile $\frac{1}{4}$ in. thick, apex angle 120° , total width 3 in., length 4 in.

8. Reproduce the roof in Question 19, p. 64, in isometric projection.
9. Draw the stone cap Question 20, p. 65, in planometric projection.
10. Reproduce the lantern roof Question 24, p. 66, in axonometric projection.
11. A grindstone 3 ft. dia., 6 in. thick, has a square hole of 4 in. side piercing it at its centre point. Draw the stone in isometric projection to a scale of 1 in. to 1 ft.
12. Draw the base of the pier in Question 6, p. 60, in (a) isometric, (b) planometric, (c) oblique projection.
13. Draw the stone cap in axonometric projection to the sizes given.
14. Assume the stone arch, Fig. 161, to measure 1 ft. 6 in. from face to back, and draw the complete arch in axonometric projection. $AE = 8$ ft. (Ref. Fig. 161.)
15. Draw an ellipse of major axis 3 in., minor axis 2 in. in oblique projection. (Ref. Fig. 157.)
16. Draw the course of brickwork at the reveal in axonometric projection to one-eighth scale.
17. Draw the herring-bone strutting in Fig. 56 in isometric projection. Scale quarter full size. (Three joists only need be shown.)
18. The joint between the lock rail and the stile of a door is given in oblique projection. Draw the rail in axonometric projection. Use your discretion for measurements not shown. Scale quarter full size.
19. Two pieces of wood of equal thickness are dovetailed together as shown. Draw the two pieces separately in isometric projection to a full size scale. The spaces between the dovetails are equal.
20. The vertical section through a trench with struts (*A*) and poling boards (*B*) is given. Assume the poling boards to be about 2 ft. 6 in. apart, and the trench 3 ft. wide and 3 ft. deep; make a freehand planometric sketch of about 9 ft. length of trench. Use your judgment in proportioning the sizes.
21. Sketch from memory a joiner's jack plane in isometric projection.

CHAPTER IX

Developments

The development of the surface of a solid is the "laying out" of the surface in one plane. E.g. Consider the simplest of the geometrical solids, the cube. This solid has six equal square faces, each of the size ABCD (Fig. 167). The development of the cube will consequently consist of six squares, joined together in such a manner that when folded or bent along certain lines, the whole will form a hollow cube. There are many arrangements for the six squares which would fulfil the above conditions, and there are many which would not conform to the rules. E.g. All the arrangements in Fig. 168 are true developments, but those in Fig. 169 are not, as under no system of folding could they be made to form cubes. For practice, develop a cube of $\frac{1}{2}$ in. edge in as many ways as possible—you will perhaps be surprised at the result.

EXAMPLE. (Figs. 170A and 170B.) *A cubical box has to be lined with zinc, the joints to be on the lines DK, JK, HK, GH. The lead has to extend to the outside of the box on the top edges. To draw the true shape of the zinc before bending.*

This is a box without lid, therefore only five squares will be needed. The base is attached to the sides along the line JG. Draw the five rectangles (each equal to the size of one of the *inside* faces of the box) as arranged in Fig. 170B, and add the strip along the top edge for the overlap on the upper edge of the box. This strip will, of course, be equal to the thickness of the box.

(*Note.*—The narrow strip on the upper edge of the development would not *completely* cover the edge of the box, the small squares at the corners (one of which is shaded at BD) still remaining uncovered.)

Prisms

EXAMPLE. (Figs. 171A and 171B.) *To develop the surface of a brick.*

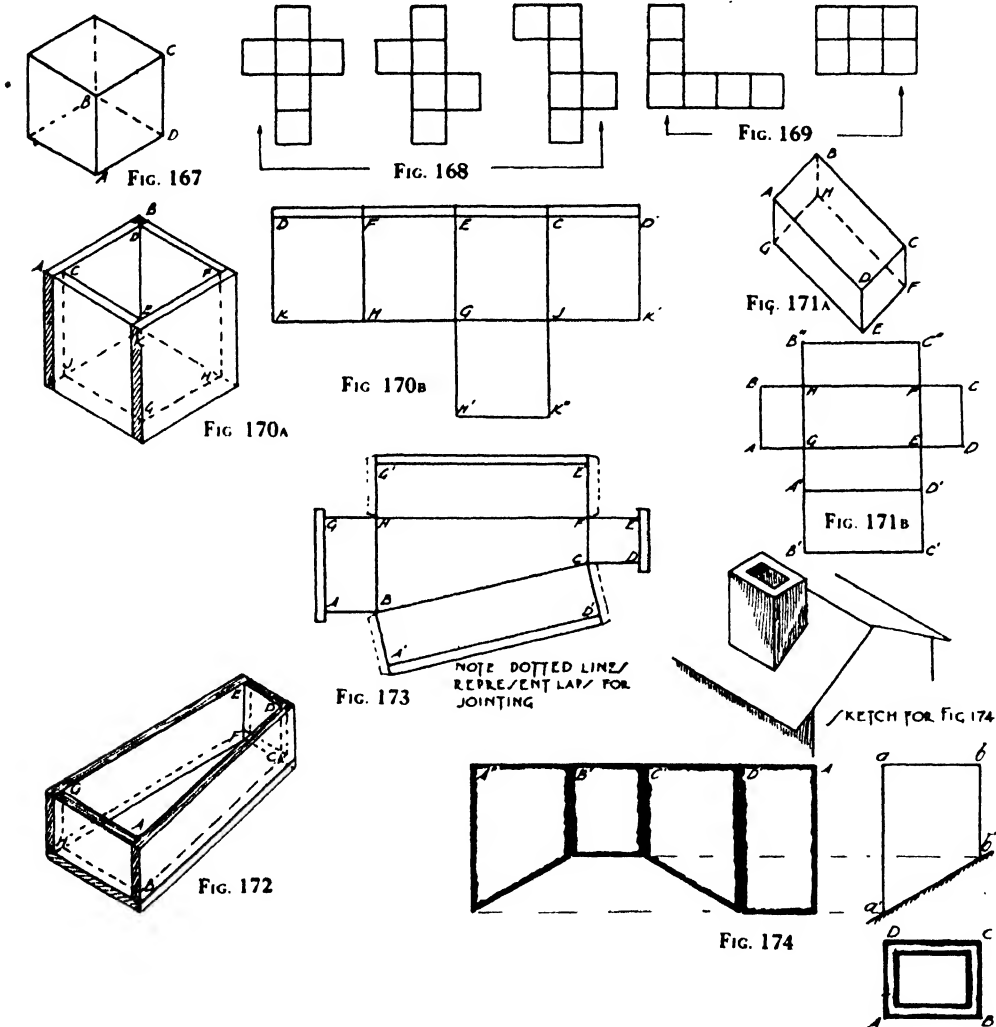
It will be plain to see that the development will consist of two rectangles each $9'' \times 4\frac{1}{2}''$, two rectangles each $4\frac{1}{2}'' \times 3''$, and two rectangles each $9'' \times 3''$. We must arrange these six rectangles, however, so that they will form a $9'' \times 4\frac{1}{2}'' \times 3''$ prism when folded along their dividing lines. One arrangement is given in Fig. 171B. The rectangles HGEF, A'B'C'D' are each $9'' \times 4\frac{1}{2}''$, rectangles GED'A', HFC''B'' are $9'' \times 3''$, and rectangles FEDC, GHBA are $4\frac{1}{2}'' \times 3''$.

EXAMPLE. (Figs. 172 and 173.) *To develop the inner surface of the zinc-lined sink, allowing 1 in. margin for bending over the top edges.*

There are many solutions of this problem according to the positions occupied by the joints. Assuming the joints are on the vertical angles, we proceed as follows :

First draw the quadrilateral HFCB. On the sides of this figure, draw rectangles each equal in width to the depth of the sink, i.e. make $FE = FE' = CD' = BA$. A 1 in. strip is now added to the top edge of each rectangle as shown. This arrangement allows for a 1 in. overlap of all corners.

EXAMPLE. (Fig. 174.) *To develop the external surface of a rectangular chimney shaft protruding through the roof.*



The development will consist of four four-sided figures—two rectangles, and two trapeziums. Much time may be saved in many development drawings if the development is projected from the appropriate orthogonal view; in this case, the elevation.

Commence by drawing one of the rectangles AD wide and aa' long; now following a clock-wise movement around the plan, we obtain the widths DC, CB, and BA. These widths are stepped off on the development at $D'C'$, $C'B'$, and $B'A''$. The development may now be completed by projecting from b' to the face $C'B'$.

EXAMPLE. (Fig. 175.) *The section ABCD and an elevation of an elbow in a ventilating flue are given. To draw the development of the three pieces forming the elbow.*

Much time may be saved by drawing the developments in the positions shown, i.e. parallel to the appropriate length of flue. The developments should require little explanation. The surfaces marked W are each equal to DC or AB in width, and those marked T equal BC or AD.

EXAMPLE. (Fig. 176.) *The section and plan of an octagonal shaft penetrating a wall are given. It is required to develop the part of the shaft marked S.*

The total width of the development A'A" will equal the distance around the section ABC . . .

Divide A'A" into eight equal parts A'B', etc., and erect perpendiculars to A'A". (These perpendiculars should be parallel to the plan of the shaft.) Now project from the points of intersection of shaft and wall to the appropriate edges on the development. You will notice that the faces GF and CB are rectangles. The construction is plainly shown and should require no further explanation.

EXAMPLE. (Fig. 177.) *Part of a square post pierced to receive a smaller square rail is given. To draw the developed surface of the post.*

Develop the complete prism making $ab = bc = AB$. The plan of the edges entering the main post at E' and F' are at EH and FG respectively. Now E enters the face AD at a distance of AE from A, and passes out of face DC at a distance of CH from C. Make $a'e = ch = cg = af = AE$. Draw perpendiculars from these points on the development to intersect horizontal lines projected from E' and F'. Now project A' and A" horizontally on to the perpendiculars from a, a', and c. Complete the development as shown.

The Cylinder

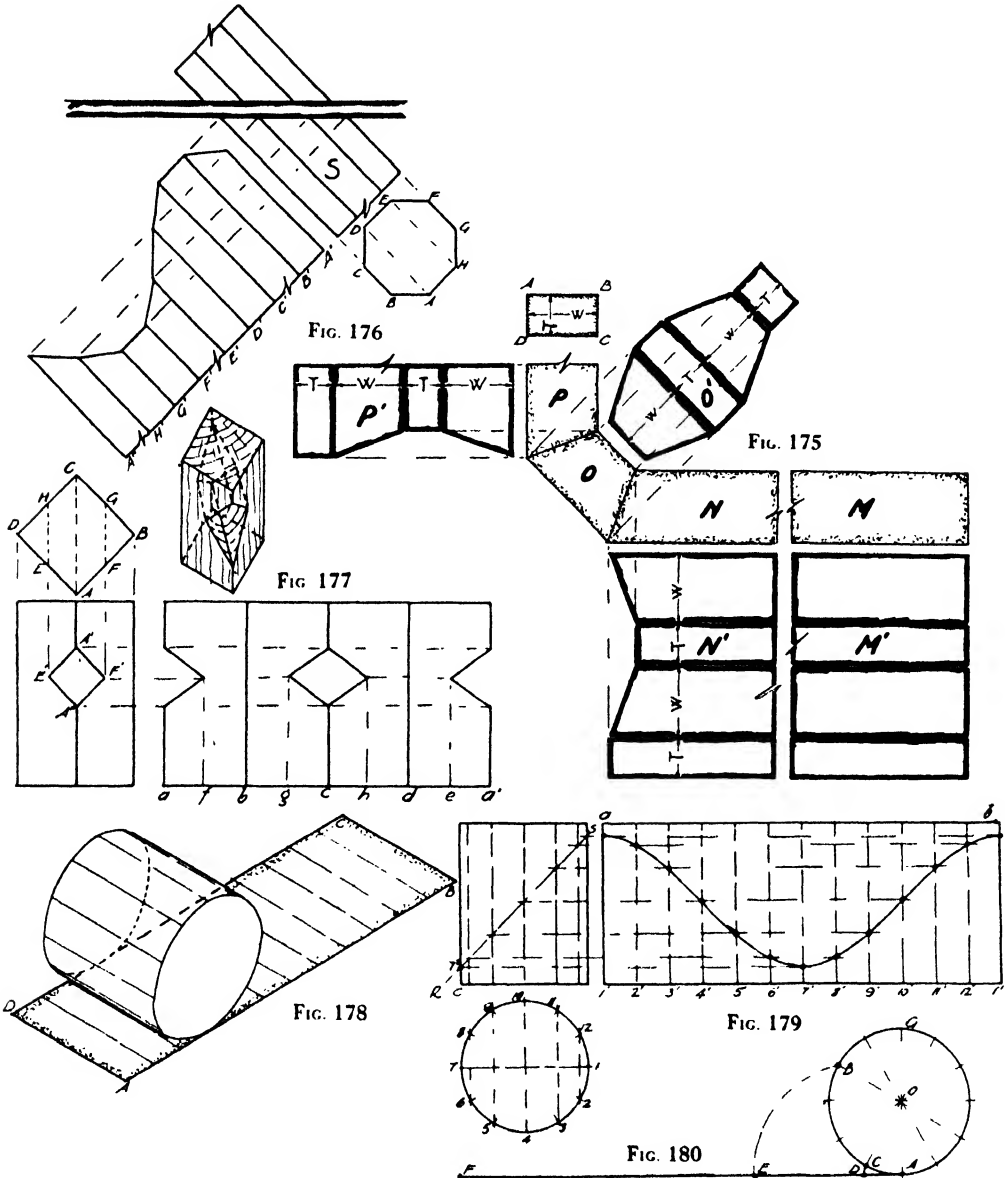
The lateral surface of a right cylinder when laid out in one plane forms a rectangle, equal in length to the circumference of the cylinder, and equal in breadth to the length of the cylinder. Thus in Fig. 178 $AB =$ the circumference, and $BC =$ the length of the cylinder. For development purposes, imagine the surface of the cylinder to be made up of a number of rectangles—12 is a convenient number—and treat the cylinder as a prism. Unless specially required, we usually omit the circular ends in the development of the cylinder. If a "complete development" is needed, the ends are included.

EXAMPLES. (Fig. 179.) (a) *To develop the surface of the pipe, the plan and elevation being given.* (b) *To develop the frustum, i.e. the portion below the cutting plane RS.*

Divide the circumference in plan into twelve equal parts (this can be done with the 60° set square) and number them 1 to 12. These points represent the plan of twelve lines drawn on the surface of the cylinder parallel to the axis. Project these lines (generators) into the elevation. The complete development is obtained by making the rectangle 1'1'ab equal in height to the cylinder, and equal in length to twelve times one of the divisions on plan. It is necessary to develop the generators 1'2'3' . . . in order to answer the second part of the problem. The distance from the base that the cutting plane RS intersects each generator is shown in the elevation, e.g. 7²c is the height from the base that RS intersects the generator 7. Project 7² to generator 7' in the development. Project all intersections across in this way, and trace a fair curve through the points thus obtained.

(Note.—The length of the circumference of a circle may be obtained mathematically,

in which case the length $1'1'$ of the development will equal the diameter of the circle in plan multiplied by π . A good method of finding the approximate length of the circumference is shown graphically in Fig. 180. Divide the circle into twelve equal parts. Draw AF per-



pendicular to one diameter, say AG. With A as centre, AC as radius, describe arc CD, and with D as centre DB as radius, draw arc BE. AE is the length of arc AB or one-third the circumference of the circle, therefore AF is three times the length AE, and the length of the circumference of the circle.)

EXAMPLE. (Fig. 181.) *Two views of a Stove pipe protruding through a roof are given. To draw the development of the two pipes forming the elbow.*

The part B is developed in a similar manner to the cylinder shown in Fig. 179. The distance L is equal to the circumference in view C. A is also developed in a similar manner. The generators in A' are drawn parallel to the axis of A for convenience, i.e. in order that the points of intersection of the generators in A and the line *cd* may be projected directly across to the development.

EXAMPLE. (Fig. 182.) *The plan, elevation, and a section of the intersection of two pipes are given. To develop parts of each pipe.*

For the development of the larger pipe, proceed as in the previous Example. Project the points of intersection of generators and circle in plan to the appropriate generators in the development. The curves obtained in this manner will be neither circles nor any other recognized curve.

The smaller pipe is developed from the elevation. Make the length *a'a'* equal to the circumference of the section, and project the points *g'h'* . . . across in the usual manner.

The Pyramid

The developments of pyramids are rather more difficult to construct than the developments of the parallel solids. Before proceeding to actual building problems, consider the square pyramid in two positions.

You will see by a reference to Fig. 183 that the development of the square pyramid consists of four equal triangles, which together equal the lateral surface of the pyramid. Now refer to Fig. 184, the triangle *V'A'B'* is *not* the true shape of the face VAB. This face is sloping away from the eye, and therefore its elevation appears to be less than it actually is. In point of fact, *V'B'* is the actual length of VE, because *V'B'* is a true edge view of VE. Now the true lengths of VB and VA have to be found before the true shape of one triangular face can be drawn.

With compass point at V radius VC swing (or "rabat") VC to a position *Vc* parallel to the vertical plane. Project *c* to *c'*; *V'c'* is the true length of VC. With centre *V'* describe an arc with *V'c'* as radius. On this arc mark off *c'd'*, *d'a'*, etc., equal in length to one edge of the base of the pyramid, say AB. Now complete the development as shown.

Fig. 185 shows a square pyramid in a different position. In this case *V'C'* is actually the true length of VC as it is a true profile view of VC. Describe an arc of radius *V'C'* with *V'* as centre, and step off around the arc *C'd*, *da*, etc., equal to AB.

Pyramids with polygons for bases are treated in a similar manner to those in the two examples above, except, of course, the development will contain as many triangles as the polygon has sides.

EXAMPLE. (Fig. 186.) *To develop a given hexagonal Pyramid.*

Ascertain the true length of one of the edges, say VA by rabatting VA to *Va*, projecting *a* to *a'*, and joining *V'a'*. Now proceed as in Fig. 184, making *a'b'* equal to AB. (Note.—Only half the development is shown.)

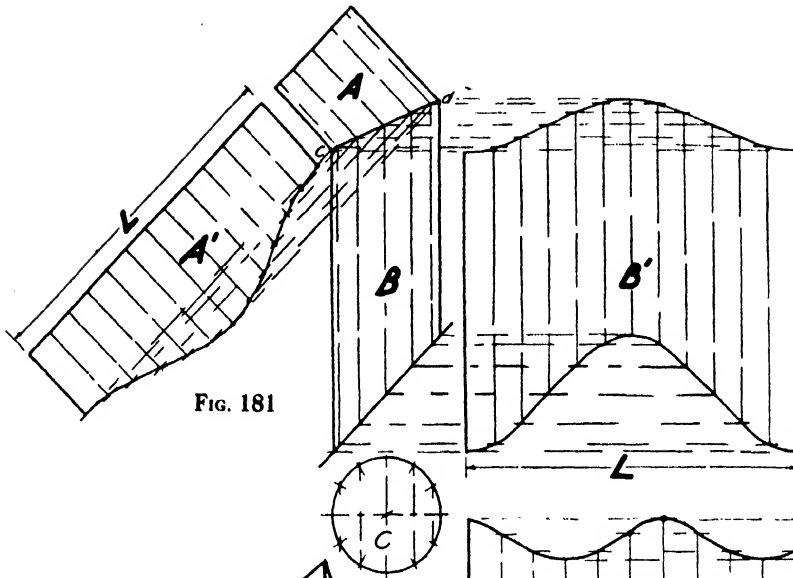
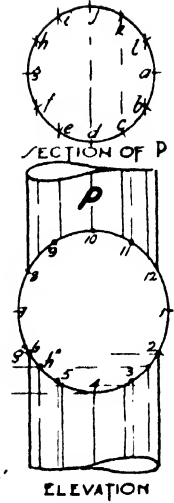


FIG. 181



ELEVATION

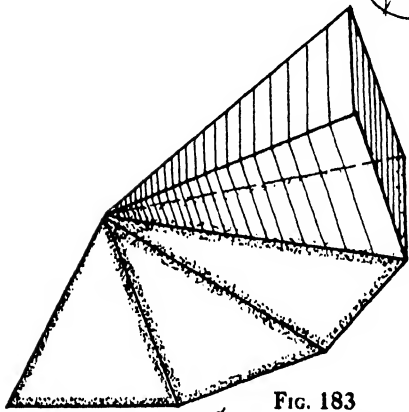


FIG. 183

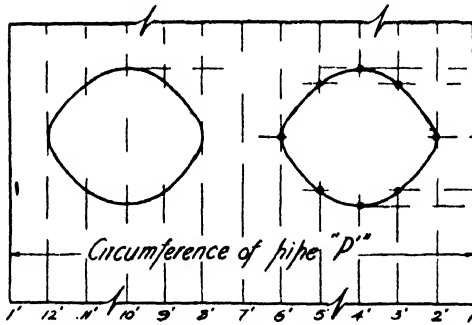
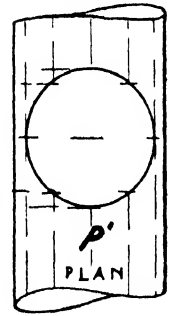


FIG. 182



PLAN

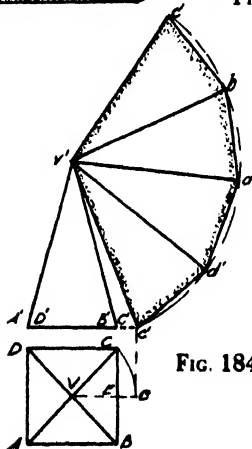


FIG. 184

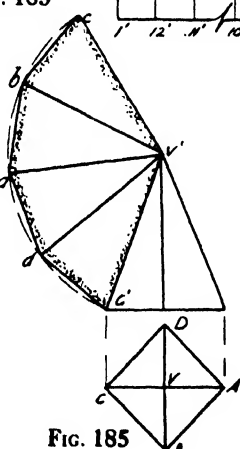


FIG. 185

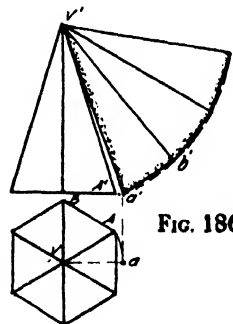


FIG. 186

EXAMPLE. (Fig. 187.) *To develop the surface of the hopper, the plan and elevation being given.*

The hopper is really the frustum of a pyramid. Continue the sides to V and V' , and develop the object as if it were a complete pyramid (Fig. 184). Now continue $E'F'$ to f' . With centre V' , radius $V'f'$ describe a second arc $f'f$. Draw the straight lines— $f'e$, eh , etc., to complete the development.

EXAMPLE. (Fig. 188.) *To develop the surface of the ventilator base protruding through the roof.*

From the given elevation form a complete pyramid as shown. Draw also the plan of this pyramid. Proceed as in the last Example until the edges in contact with the roof have to be developed. Project L' to l , and with V' as centre describe the arc ll' . Draw lm , mc' , $c'd'$ and $d'l'$.

EXAMPLE. (Fig. 189.) *The front elevation and section of an octagonal window lining are given. It is required to develop the lining.*

Continue cm to intersect $a'n$ at V' . This represents a half pyramid with V' as vertex. With V' as centre radii $V'c$ and $V'm$, describe arcs sufficiently long to accommodate four chords each equal in length to AB . Complete the development as shown.

The Cone

If the cone is regarded as a circular pyramid, little difficulty will arise in its development. The development is a sector of a circle. (Figs. 190 and 191.)

EXAMPLE. (Fig. 191.) *To develop the conical roof of the turret.*

Divide the plan into any convenient number of parts (say twelve). With centre V' and radius $V'1'$ describe arc $1'1_1$, equal in length to the circumference in plan, i.e. twelve times $1'2$. The sector $V'1_11'$ is the development.

EXAMPLE. (Figs. 192 and 192A.) *To develop the surface of a conical turret protruding through the ridge of a roof.*

Develop the cone as if it were a complete solid as explained in the previous Example.

Now draw the plan and elevation of the generators, numbering them as shown in plan, elevation, and development. At the points of intersection between the roof surfaces and the generators in elevation, project horizontally to the line $V'1'$. With compass point on V' rabat these points on $V'1'$ to the corresponding generators in the development. E.g. The highest point in the roof intersects the generators $4'$ and $10'$. Project this point to P , and with radius $V'P$, centre V' , rabat P to generators 4_1 and 10_1 in the development. Treat all the points on $V'1'$ in this manner, and trace a fair curve through the points thus obtained on the development.

EXAMPLE. (Fig. 193.) *To develop the splayed lining around the head of a semicircular window opening.*

Divide the outer edge of the lining into any convenient number of parts (say eight). With centre V' on the springing line, radii $V'E'$ and $V'e'$ describe arcs $E'F'$ and $e'f'$, making $E'F'$ equal in length to the outside edge of the lining, i.e. AEB or eight times AD . Join $F'V'$. The shaded portion is the true shape of the lining prior to its being bent to shape.

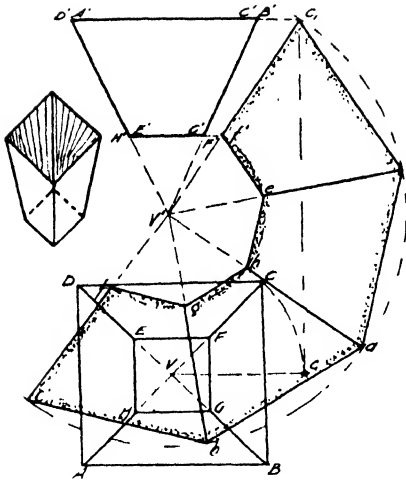


FIG. 187

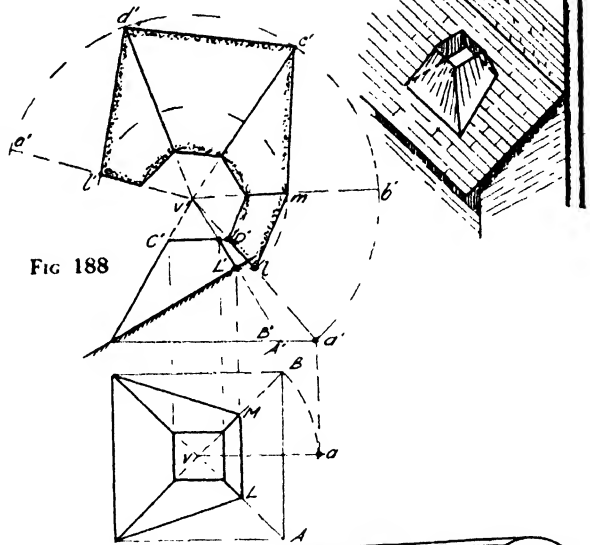


FIG. 188

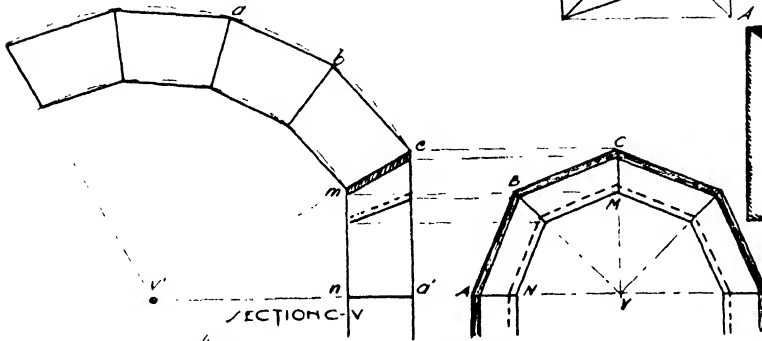


FIG. 189

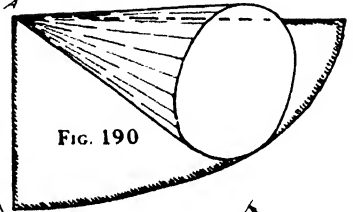


FIG. 190

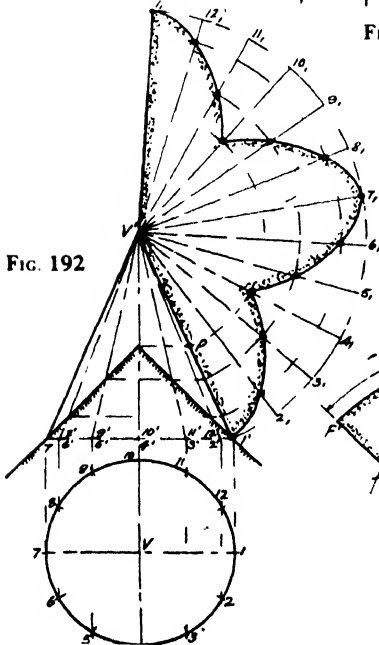


FIG. 192

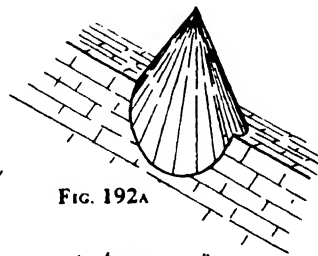


FIG. 192A

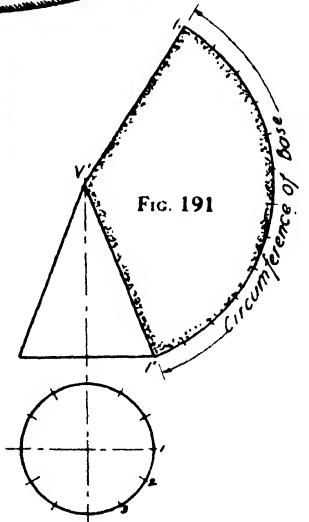


FIG. 191

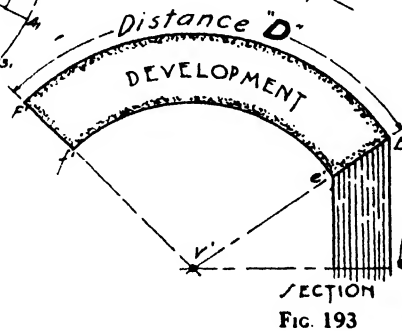


FIG. 193

Simple Miscellaneous Developments

The soffit of a semicircular skew arch. (Fig. 194.)

This is a part of a cylinder. From the elevation, project the joint lines of the voussoirs on the soffit on to the plan. W has to equal the length of the soffit or intrados, so the joint lines in elevation can be utilized for this purpose. Project horizontally the joint lines in plan across to the generators in the development as shown.

The surface of a wedge. (Fig. 195.)

Re-draw the front face ABE at *abe*. The adjoining surface BCEF is a rectangle, the development of which is shown at *bcfe*. A repetition of these two diagrams at *cdf* and *da,e,f* completes the development with the exception of the top face ABCD. This rectangle may be inserted in the development in any appropriate position.

The surface of a hipped roof. (Fig. 196.)

With compass point at B', rabat point F' to *f*. Project *f* to *f'*. CB*f'* is the true shape of the hipped surface.

Rabat *f'* to *f''* with centre B and complete as shown. The two remaining surfaces are similar, and may be copied from those obtained.

A metal lining for a sink. The joints to be between the sides and ends. (Fig. 197.)

ABCDEF is the plan of the undeveloped lining. Rabat the surface EBCF on to the horizontal, by making E'*c'* equal to E'B', and projecting *c'* to *c* and *b*. FcbE is the development of the sloping side of the lining. Reproduce the elevation at DF*fd* and at AE*ea*. Make Aa' equal Aa and complete the development as shown.

EXERCISES

1. A rectangular box (without lid) 2' 0" × 1' 0" × 9" (depth) has to be lined with zinc which must be in one piece. The joints have to be at the vertical angles. Develop the lining. Scale 1 in. to 1 ft. (Ref. Fig. 173.)

2. The elevation and section of a ventilating-pipe elbow protruding through a roof are given: the pipes have the same cross-section. Assume the roof to be pitched at 30°, and develop both pipes. Scale one-eighth full size. (Ref. Fig. 174.)

3. A small cardboard model of a building has to be made in one piece to the sizes given. Set out the true shape of the card before folding, showing the position of the laps for gumming purposes.

4. The plan and elevation of the lead lining for a sink is shown. Develop the lining to a scale of 1 in. to 1 ft. (Ref. Fig. 173.)

5. An octagonal post, the part elevation of which is given, is pierced by a square hole to receive a rail of 3 in. square section.

(a) Develop the surface of the post.

(b) Develop the octagonal cap.

Scale quarter full size. (Ref. Figs. 177 and 186.)

6. Assume the two pipes forming the elbow in Question 2 to be circular in cross-section, and develop both parts: the diameter of the pipes to be 6 in. Scale quarter full size. (Ref. Fig. 181.)

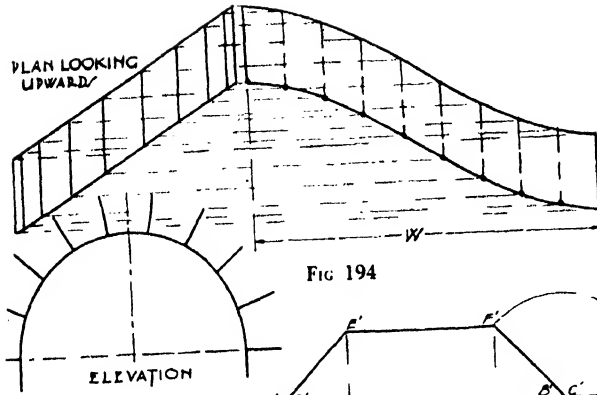


FIG 194

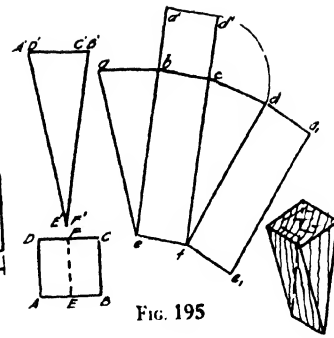


FIG. 195

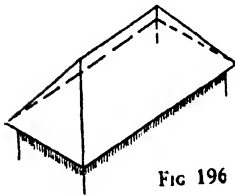


FIG 196

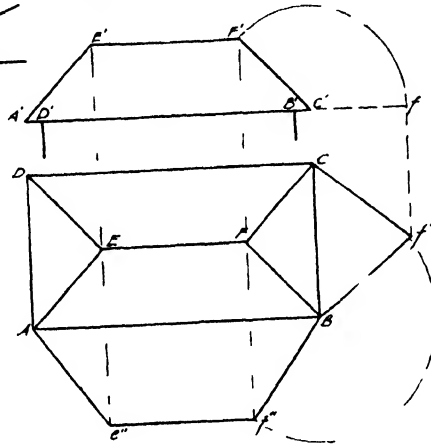
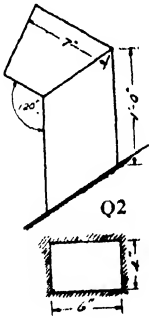
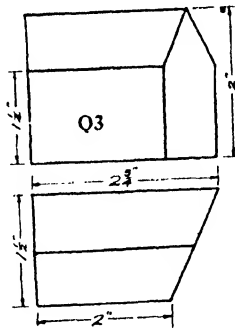


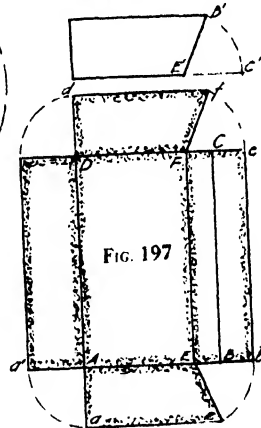
FIG. 197



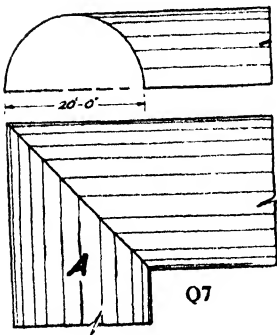
Q2



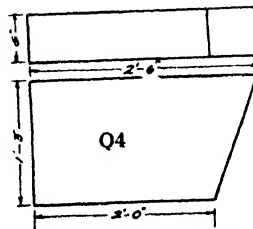
Q3



Q4



Q7



Q5

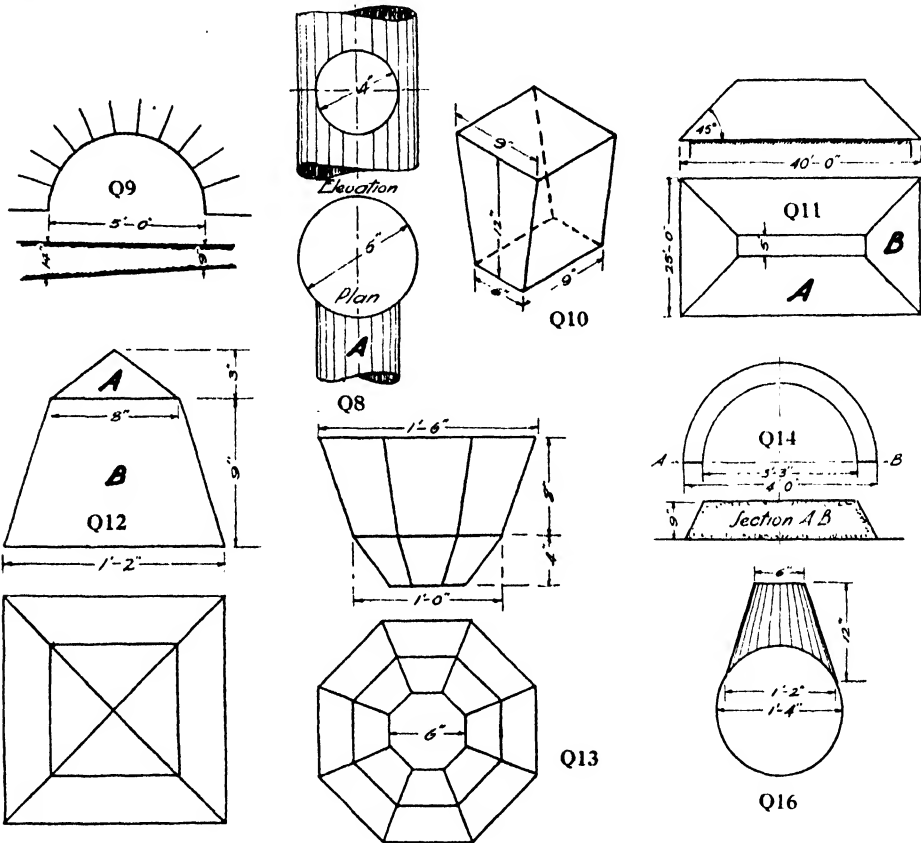
7. Two semicircular roof surfaces intersect as shown. Develop the surface marked A. Scale 1 in. to 10 ft. (Ref. Fig. 179.)

8. Develop the portion of the branch pipe (marked A) to a scale of half full size. (Ref. Fig. 182.)

9. The plan and elevation of a semicircular opening in a tapering wall are given. Develop the soffit of the arch to a scale of $\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 194.)

10. Develop the six surfaces of the keystone as one templet. Scale $1\frac{1}{2}$ in. to 1 ft. (Ref. Fig. 195.)

11. Develop the two surfaces of the roof marked A and B. Scale 1 in. to 10 ft. (Ref. Fig. 196.)



12. Develop the surfaces of the two parts A and B of the stone cap. Treat each part separately. Scale one-eighth full size. (Note: The cap consists of a complete square pyramid, and the frustum of a square pyramid.) (Ref. Figs. 184 and 187.)

13. The diagrams represent the plan and elevation of a lamp shade. Develop the two parts separately. Scale $1\frac{1}{2}$ in. to 1 ft.

14. The plan and elevation of a splayed lining around a window opening are shown in the diagram. Develop the part of the lining above the springing line AB. Scale $\frac{3}{4}$ in. to 1 ft. (Ref. Fig. 193.)

15. If the lamp shade in Exercise 12 consisted of double cones instead of pyramids, what would the shape of the developments be? Draw the developments using the given dimensions, to a scale of 1 in. to 1 ft.

16. Develop the part of the cone protruding above the cylindrical boiler.

